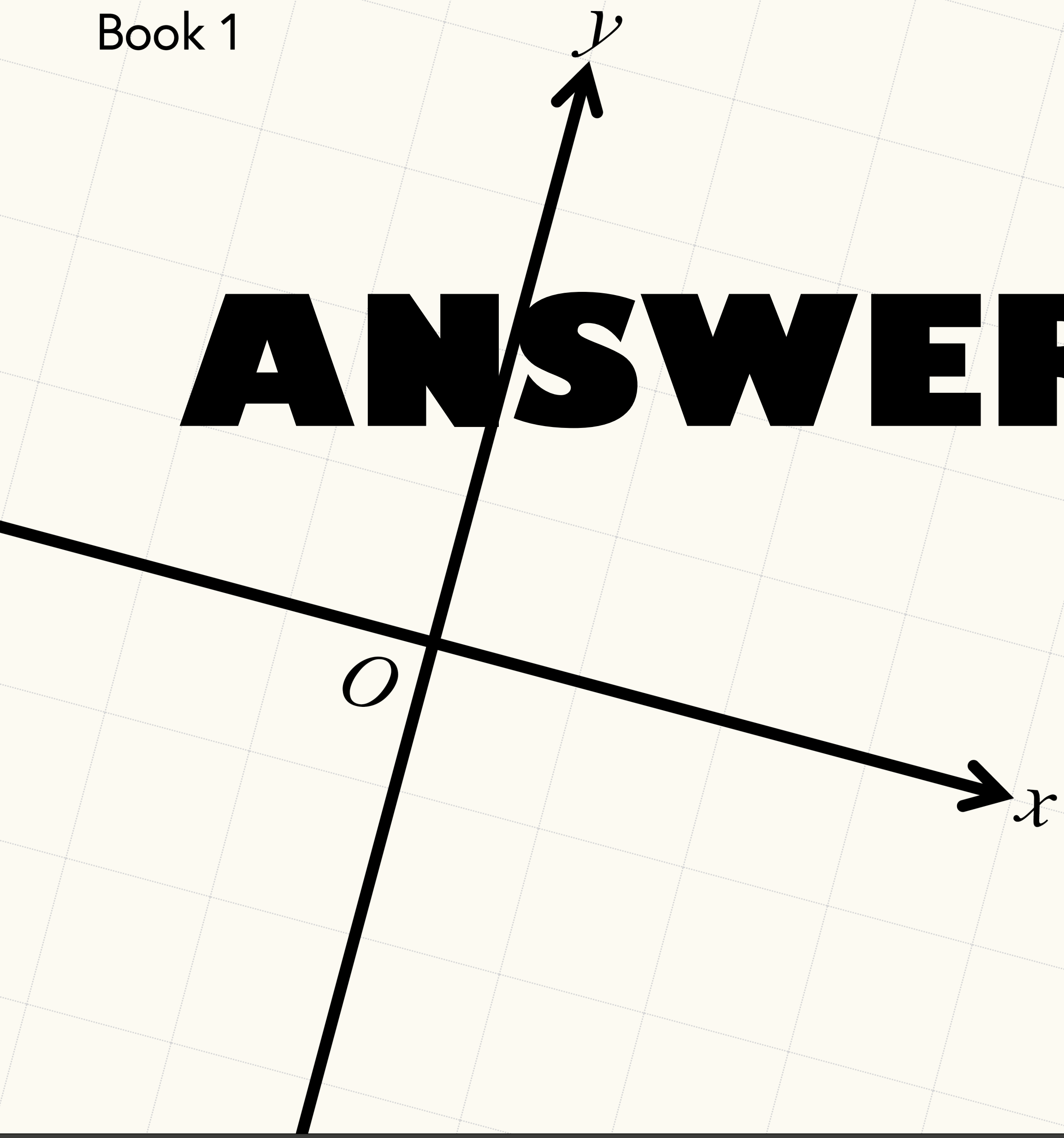
YOUR PRACTICE SET

ANALYSIS AND APPROACHES FOR IBDP MATHEMATICS

Book 1



ANSWERS

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- Common Topics for both SL and HL students
- 100 Examples + 400 Intensive Exercises
- 375 Short Questions + 125 Structured Questions
- Skills on GDC

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Chapter 1 Solution

Exercise 1

1. (a) The required circumference

$= 1730 \times \pi$ (M1) for correct formula

$= 5434.955291$

$= 5.43 \times 10^3$ cm A1 N2

[2]

(b) The required area

$= \left(\frac{1730}{2}\right)^2 \times \pi$ (M1) for correct formula

$= 2350618.163$

$= 2.35 \times 10^6$ cm^2 A1 N2

[2]

2. (a) The required length of hypotenuse

$= \sqrt{3348^2 + 14880^2}$ (M1) for correct formula

$= 15252$

$= 1.53 \times 10^4$ cm A1 N2

[2]

(b) The required area

$= \frac{1}{2} \times 3348 \times 14880$ (M1) for correct formula

$= 24909120$

$= 2.49 \times 10^7$ cm^2 A1 N2

[2]

3. (a) The required height

$= \frac{22489932}{5476}$ (M1) for correct formula

$= 4107$

$= 4.11 \times 10^3$ cm A1 N2

[2]

(b) The required length of diagonal

$= \sqrt{4107^2 + 5476^2}$ (M1) for correct formula

$= 6845$

$= 6.85 \times 10^3$ cm A1 N2

[2]

4. (a) The required base length

$= \frac{331320000}{8283} \times 2$ (M1) for correct formula

$= 80000$

$= 8 \times 10^4$ cm A1 N2

[2]

(b) The required length of hypotenuse

$= \sqrt{80000^2 + 8283^2}$ (M1) for correct formula

$= 80427.65749$

$= 8.04 \times 10^4$ cm A1 N2

[2]

Chapter 2 Solution

Exercise 2

1. (a) $f(x)=0$ — (M1) for setting equation

$x^2-6x+8=0$

$(x-2)(x-4)=0$ — A1

$x=2$ or $x=4$

Hence, the x-intercepts are 2 and 4 respectively. — A2 N2

[4]

(b) (i) $x=3$ — A1 N1

(ii) The y-coordinate of the vertex

$=3^2-6(3)+8$ — (M1) for substitution

$=-1$ — A1 N2

[3]

2. (a) $f(x)=0$ — (M1) for setting equation

$x^2-11x+10=0$

$(x-10)(x-1)=0$ — A1

$x=10$ or $x=1$

Hence, the x-intercepts are 1 and 10 respectively. — A2 N2

[4]

(b) (i) $x=5.5$ — A1 N1

(ii) The y-coordinate of the vertex

$=5.5^2-11(5.5)+10$ — (M1) for substitution

$=-20.25$ — A1 N2

[3]

3. (a) $f(x)=0$ — (M1) for setting equation

$-2x^2-14x=0$

$-2x(x+7)=0$ — A1

$x=0$ or $x=-7$

Hence, the x-intercepts are 0 and -7 respectively. — A2 N2

[4]

(b) (i) $x=-3.5$ — A1 N1

(ii) The y-coordinate of the vertex

$=-2(-3.5)^2-14(-3.5)$ — (M1) for substitution

$=24.5$ — A1 N2

[3]

4. (a) $f(x) = 0$ (M1) for setting equation

$13.5 - 1.5x^2 = 0$

$1.5(9 - x^2) = 0$ A1

$1.5(3 + x)(3 - x) = 0$

$x = -3$ or $x = 3$

Hence, the x-intercepts are -3 and 3 respectively. A2 N2

[4]

(b) (i) $x = 0$ A1 N1

(ii) The y-coordinate of the vertex

$= 13.5 - 1.5(0)^2$ (M1) for substitution

$= 13.5$ A1 N2

[3]

Exercise 3

1.	(a)	$x=-5$ and $x=7$	A2	N2	
					[2]
	(b)	$h=\frac{-5+7}{2}$	(M1) for correct formula		
		$h=1$	A1		
		$k=(1-7)(1+5)$	(M1) for finding k		
		$k=-36$			
		Therefore, the coordinates of the vertex are $(1,-36)$.	A1	N3	
					[4]
2.	(a)	$x=-1$ and $x=-6$	A2	N2	
					[2]
	(b)	$h=\frac{-6-1}{2}$	(M1) for correct formula		
		$h=-\frac{7}{2}$	A1		
		$k=2\left(-\frac{7}{2}+1\right)\left(-\frac{7}{2}+6\right)$	(M1) for finding k		
		$k=-\frac{25}{2}$			
		Therefore, the coordinates of the vertex are $\left(-\frac{7}{2},-\frac{25}{2}\right)$	A1	N3	
					[4]
3.	(a)	$p=5$ and $q=11$	A2	N2	
					[2]
	(b)	$x=8$	A1	N1	
					[1]
	(c)	$-7.5=a(10-5)(10-11)$	M1A1		
		$-7.5=-5a$			
		$a=1.5$	A1	N2	
					[3]
4.	(a)	$p=0$ and $q=18$	A2	N2	
					[2]
	(b)	$x=9$	A1	N1	
					[1]
	(c)	$30=a(0-15)(15-18)$	M1A1		
		$30=45a$			
		$a=\frac{2}{3}$	A1	N2	
					[3]

Exercise 4

1. $\Delta = b^2 - 4ac$ (M1) for discriminant

$b^2 - 4ac = 0$ R1

$(-5)^2 - 4(1)(k^2) = 0$ (A1) for substitution

$25 - 4k^2 = 0$

$(5+2k)(5-2k) = 0$ (M1)A1 for factorizing

$k = -\frac{5}{2}$ or $k = \frac{5}{2}$ A2 N4

[7]

2. $\Delta = b^2 - 4ac$ (M1) for discriminant

$b^2 - 4ac > 0$ R1

$(4k)^2 - 4(1)(2k) > 0$ (A1) for substitution

$16k^2 - 8k > 0$ A1

$8k(2k-1) > 0$ (M1) for factorizing

$k < 0$ or $k > \frac{1}{2}$ A2 N4

[7]

3. $\Delta = b^2 - 4ac$ (M1) for discriminant

$b^2 - 4ac < 0$ R1

$(k-1)^2 - 4(1)(1) < 0$ (M1)(A1) for substitution

$k^2 - 2k - 3 < 0$ A1

$(k+1)(k-3) < 0$ (M1) for factorizing

$-1 < k < 3$ A2 N4

[8]

4. $\Delta = b^2 - 4ac$ (M1) for discriminant

$b^2 - 4ac \geq 0$ R1

$(4k+16)^2 - 4(4)(25k) \geq 0$ (M1)(A1) for substitution

$16k^2 - 272k + 256 \geq 0$ A1

$16(k-1)(k-16) \geq 0$ (M1) for factorizing

$k \leq 1$ or $k \geq 16$ A2 N4

[8]

Exercise 5

1. (a) $x=-2$ is one of the x-intercepts. (M1) for valid approach

$\frac{p-2}{2}=1$ (M1) for correct formula

$p=4$ A1 N2

[3]

(b) $-32=a(0-4)(0+2)$ (M1)(A1) for substitution

$-32=-8a$

$a=4$ A1 N2

[3]

(c) A tangent only intersects with a curve once. (M1) for correct property

Thus the discriminant for $f(x)=4mx-57$

equals to 0. R1

$4mx-57=4(x-4)(x+2)$ (M1) for setting equation

$4mx-57=4x^2-8x-32$

$4x^2-(4m+8)x+25=0$ (M1) for quadratic equation

$(4m+8)^2-4(4)(25)=0$ A1

$16m^2+64m-336=0$

$16(m-3)(m+7)=0$ (A1) for factorization

$m=3$ or $m=-7$ A2 N0

[8]

2. (a) $x=4$ is one of the x-intercepts. (M1) for valid approach

$\frac{q+4}{2}=2.5$ (M1) for correct formula

$q=1$ A1 N2

[3]

(b) $-4=a(5-4)(5-1)$ (M1)(A1) for substitution

$-4=4a$

$a=-1$ A1 N2

[3]

(c) A tangent only intersects with a curve once. (M1) for correct property

Thus the discriminant for $f(x)=mx$ equals to 0. R1

$mx=-(x-4)(x-1)$ (M1) for setting equation

$mx=-x^2+5x-4$

$x^2+(m-5)x+4=0$ (M1) for quadratic equation

$(m-5)^2-4(1)(4)=0$ A1

$m^2-10m+9=0$

$(m-1)(m-9)=0$ (A1) for factorization

$m=1$ or $m=9$ A2 N0

[8]

3.	(a)	$12 = (3 - p)(3 - 1)$	(M1)(A1) for correct formula
		$6 = 3 - p$	
		$p = -3$	A1 N2
			[3]
	(b)	Recognizing 1 and -3 are the x-intercepts	(M1) for valid approach
		The x-coordinate of the vertex	
		$= \frac{1-3}{2}$	(M1) for substitution
		$= -1$	A1 N3
			[3]
	(c)	A tangent only intersects with a curve once.	(M1) for correct property
		Thus the discriminant for $f(x) = m(x-1)$ equals to 0.	R1
		$m(x-1) = (x+3)(x-1)$	(M1) for setting equation
		$mx - m = x^2 + 2x - 3$	
		$x^2 + (2-m)x + (m-3) = 0$	(M1) for quadratic equation
		$(m-2)^2 - 4(1)(m-3) = 0$	A1
		$m^2 - 8m + 16 = 0$	
		$(m-4)(m-4) = 0$	(A1) for factorization
		$m = 4$	A2 N0
			[8]

4. (a) $-9 = a(0-p)(0+p)$ M1

$-9 = a(-p)(p)$ A1

$-9 = -ap^2$

$a = \frac{9}{p^2}$ AG N0

[2]

(b) $-5 = a(1-p)(1+p)$ A1

$-5 = a(1-p^2)$

$\therefore -5 = \left(\frac{9}{p^2}\right)(1-p^2)$ (M1) for substitution

$-5p^2 = 9 - 9p^2$

$4p^2 = 9$

$p^2 = \frac{9}{4}$

$p = -1.5$ (*Rejected*) or $p = 1.5$ A1

$a = \frac{9}{1.5^2}$

$a = 4$ A1 N2

[4]

(c) A tangent only intersects with a curve once. (M1) for correct property

Thus the discriminant for

$f(x) = -4mx - (9+m)$ equals to 0. R1

$-4mx - (9+m) = 4(x-1.5)(x+1.5)$ (M1) for setting equation

$-4mx - (9+m) = 4x^2 - 9$

$4x^2 + 4mx + m = 0$ (M1) for quadratic equation

$(4m)^2 - 4(4)(m) = 0$ A1

$16m^2 - 16m = 0$

$16m(m-1) = 0$ (A1) for factorization

$m = 0$ or $m = 1$ A2 N0

[8]

Exercise 6

1.	$-x^2-4x=2kx+1$	(M1) for setting equation
	$x^2+(2k+4)x+1=0$	A1
	$\Delta=b^2-4ac$	(M1) for discriminant
	$b^2-4ac<0$	R1
	$(2k+4)^2-4(1)(1)<0$	(A1) for substitution
	$4k^2+16k+16-4<0$	
	$4k^2+16k+12<0$	
	$4(k+3)(k+1)<0$	(M1) for factorization
	$-3<k<-1$	A2 N3
		[8]
2.	$x^2-4x-4k=2kx-16$	(M1) for setting equation
	$x^2-(2k+4)x+(16-4k)=0$	A1
	$\Delta=b^2-4ac$	(M1) for discriminant
	$b^2-4ac>0$	R1
	$(2k+4)^2-4(1)(16-4k)>0$	(A1) for substitution
	$4k^2+16k+16-64+16k>0$	
	$4k^2+32k-48>0$	
	$k^2+8k-12>0$	
	$k^2+8k+16>28$	
	$(k+4)^2>28$	(M1) for factorization
	$k+4<-\sqrt{28}$ or $k+4>\sqrt{28}$	
	$k<-9.29$ or $k>1.29$	A2 N3
		[8]
3.	$x^2-1.5k=(8-k)x-16$	(M1) for setting equation
	$x^2+(k-8)x+(16-1.5k)=0$	A1
	$\Delta=b^2-4ac$	(M1) for discriminant
	$b^2-4ac\geq 0$	R1
	$(k-8)^2-4(1)(16-1.5k)\geq 0$	(A1) for substitution
	$(k-8)^2-4(1)(16-1.5k)\geq 0$	
	$k^2-16k+64-64+6k\geq 0$	
	$k^2-10k\geq 0$	
	$k(k-10)\geq 0$	(M1) for factorization
	$k\leq 0$ or $k\geq 10$	A2 N3
		[8]

4.	$x^2 + 2x - 2k = 9 - kx$	(M1) for setting equation
	$x^2 + (k+2)x - (2k+9) = 0$	A1
	$\Delta = b^2 - 4ac$	(M1) for discriminant
	$b^2 - 4ac \le 0$	R1
	$(k+2)^2 + 4(1)(2k+9) \le 0$	(A1) for substitution
	$k^2 + 4k + 4 + 8k + 36 \le 0$	
	$k^2 + 12k + 40 \le 0$	(M1) for simplification
	$k^2 + 12k + 36 \le -4$	
	$(k+6)^2 \le -4$	(M1) for factorization
	Therefore, there is no real solution for *k*.	A1 N3

[8]

Chapter 3 Solution

Exercise 7

1. (a)

$y = 8x - 1$

$\Rightarrow x = 8y - 1$ (M1) for swapping variables

$8y = x + 1$

$y = \frac{x+1}{8}$ (A1) for changing subject

$\therefore f^{-1}(x) = \frac{x+1}{8}$ A1 N2

[3]

(b)

$g(5)$

$= 5^2 - 5$ (M1) for substitution

$= 20$

$(f \circ g)(5)$

$= f(20)$

$= 8(20) - 1$ (A1) for substitution

$= 159$ A1 N3

[3]

2. (a)

$y = 2x - 3$

$\Rightarrow x = 2y - 3$ (M1) for swapping variables

$2y = x + 3$

$y = \frac{x+3}{2}$ (A1) for changing subject

$\therefore f^{-1}(x) = \frac{x+3}{2}$ A1 N2

[3]

(b)

$f(-2)$

$= 2(-2) - 3$ (M1) for substitution

$= -7$

$(g \circ f)(-2)$

$= g(-7)$

$= (-7 + 5)^2$ (A1) for substitution

$= 4$ A1 N3

[3]

3. (a) $y = \sqrt{x+4}$

$\Rightarrow x = \sqrt{y+4}$ (M1) for swapping variables

$4 = \sqrt{y+4}$

$16 = y+4$

$y = 12$ (M1) for valid approach

$\therefore f^{-1}(4) = 12$ A1 N2

[3]

(b) $g(96) = 7$

$\Rightarrow g^{-1}(7) = 96$ (M1) for valid approach

$(f \circ g^{-1})(7)$

$= f(96)$

$= \sqrt{96+4}$ (A1) for substitution

$= 10$ A1 N3

[3]

4. (a) $y = \sqrt{2x-1}$

$\Rightarrow x = \sqrt{2y-1}$ (M1) for swapping variables

$3 = \sqrt{2y-1}$

$9 = 2y-1$

$y = 5$ (M1) for valid approach

$\therefore f^{-1}(3) = 5$ A1 N2

[3]

(b) $g\left(\frac{3a+1}{2}\right) = 2$

$\Rightarrow g^{-1}(2) = \frac{3a+1}{2}$ (M1) for valid approach

$(f \circ g^{-1})(2)$

$= f\left(\frac{3a+1}{2}\right)$

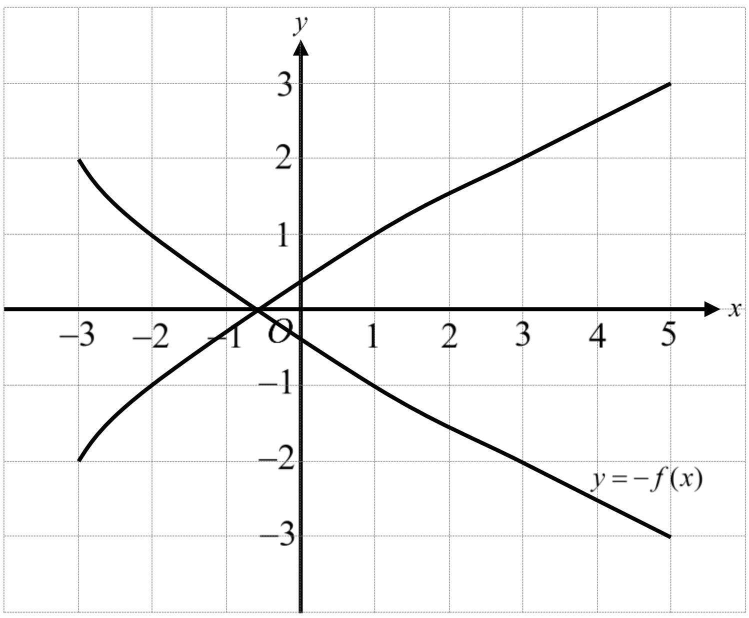
$= \sqrt{2\left(\frac{3a+1}{2}\right) - 1}$ (A1) for substitution

$= \sqrt{3a}$ A1 N3

[3]

Exercise 8

1.	(a)	$f(-3) = -2$	(M1) for valid approach	
		$\therefore f^{-1}(-2) = -3$	A1 N2	
				[2]
	(b)	$f(5) = 3$	(M1) for valid approach	
		$(f \circ f)(5)$		
		$= f(3)$	(A1) for composite function	
		$= 2$	A1 N3	
				[3]
	(c)	For correct y-intercept	A1	
		For any two correct points from $(-3, 2)$, $(3, -2)$ and $(5, -3)$	A1 N2	
				[2]



2. (a) $f(0)=2$ (M1) for valid approach

$\therefore f^{-1}(2)=0$ A1 N2

[2]

(b) $f(4)=0$ (M1) for valid approach

$(f \circ f)(4)$

$= f(0)$ (A1) for composite function

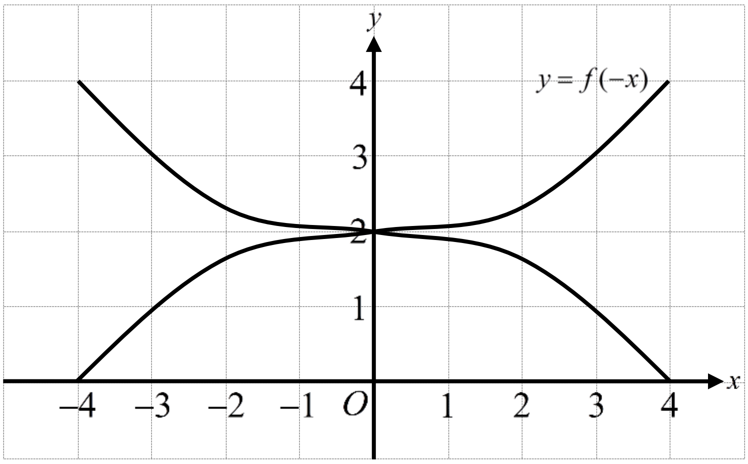
$= 2$ A1 N3

[3]

(c) For correct y-intercept A1

For correct points from $(4, 4)$, $(0, 2)$ and $(-4, 0)$ A1 N2

[2]



3. (a) $-2 \le x \le 3$ A2 N2

[2]

(b) $f^{-1}(1) = -2$ (M1) for valid approach

$(f^{-1} \circ f^{-1})(1)$

$= f^{-1}(-2)$ (A1) for composite function

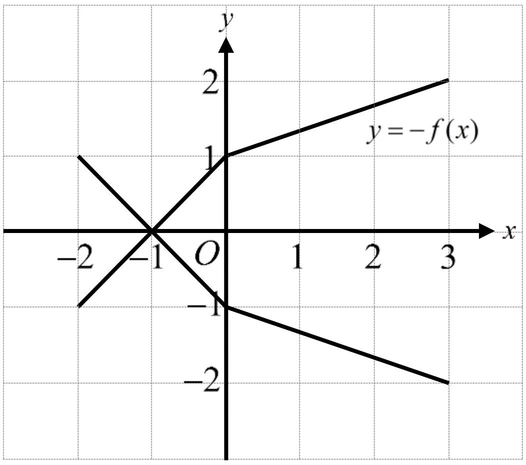
$= 3$ A1 N3

[3]

(c) For correct y-intercept A1

For any two correct points from $(-2, -1)$, $(0, 1)$ and $(3, 2)$ A1 N2

[2]



4. (a) $-4 \le x \le 3$ A2 N2

[2]

(b) $f^{-1}(3) = -4$ (M1) for valid approach

$(f^{-1} \circ f^{-1})(3)$

$= f^{-1}(-4)$ (A1) for composite function

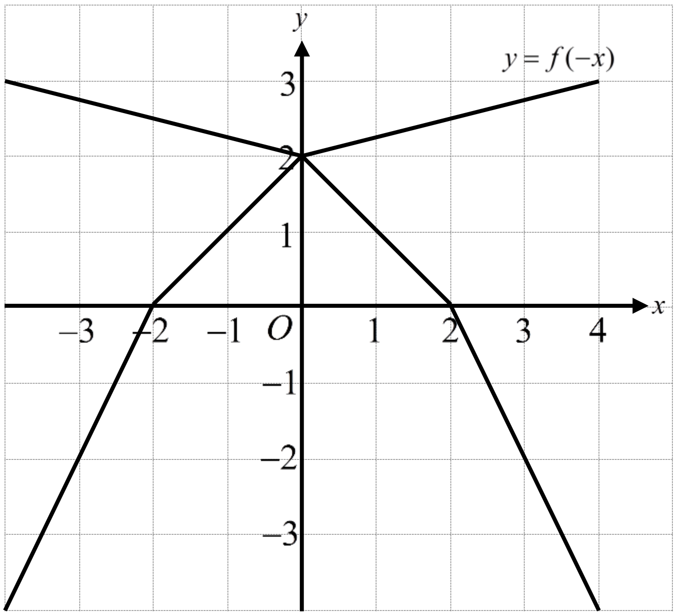
$= 4$ A1 N3

[3]

(c) For correct y-intercept A1

For correct points $(4, 3)$ and $(-4, -4)$ A1 N2

[2]



Exercise 9

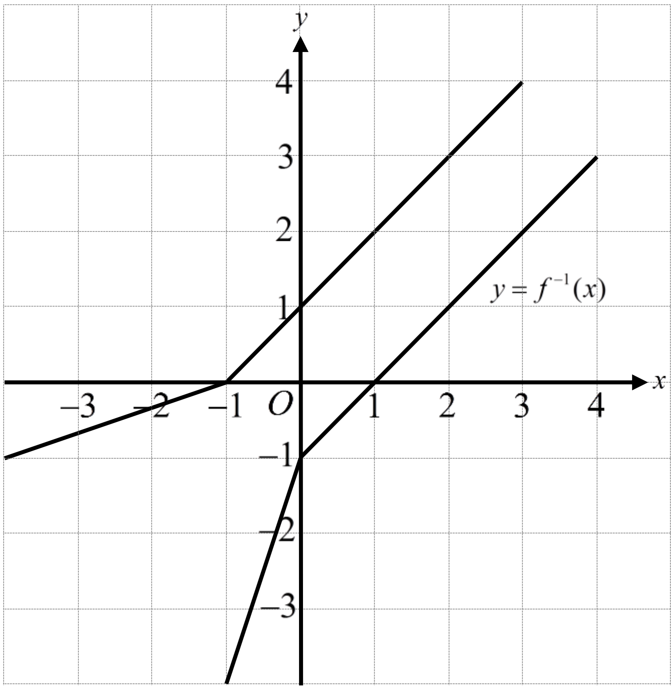
1.	(a)	(i)	$f(2)=3$	A1	N1
		(ii)	$f^{-1}(-1)=-4$	A2	N2

[3]

(b) For any two correct points from $(-1,-4)$, $(0,-1)$ or $(4,3)$ M1

For correct graph A2 N3

[3]



2. (a) (i) $f(-4) = 3$ A1 N1

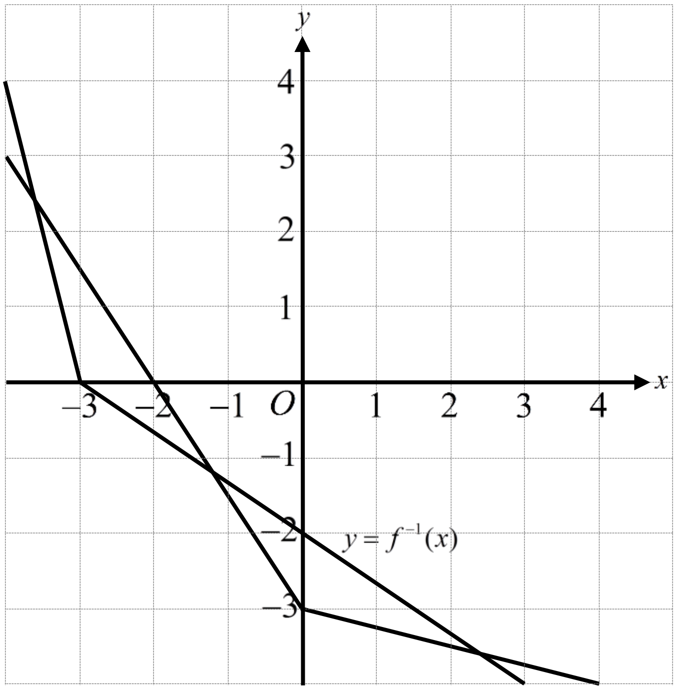
(ii) $f^{-1}(-4) = 4$ A2 N2

[3]

(b) For any two correct points from $(-4, 4)$, $(-3, 0)$ or $(3, -4)$ M1

For correct graph A2 N3

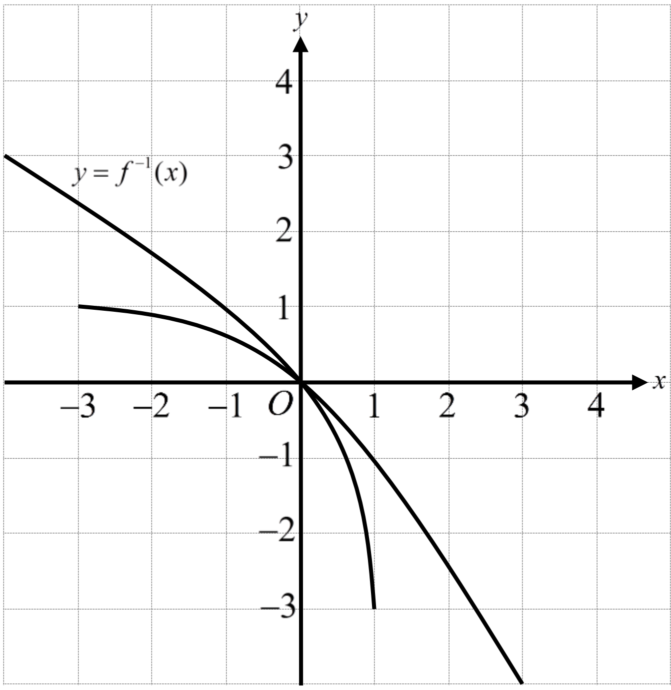
[3]



3. (a) For any two correct points from $(-4, 3)$, $(0, 0)$ or $(1, -3)$ M1

For correct graph A2 N3

[3]



(b) The required coordinates for B

$= (1-1, 2(-1))$ M2

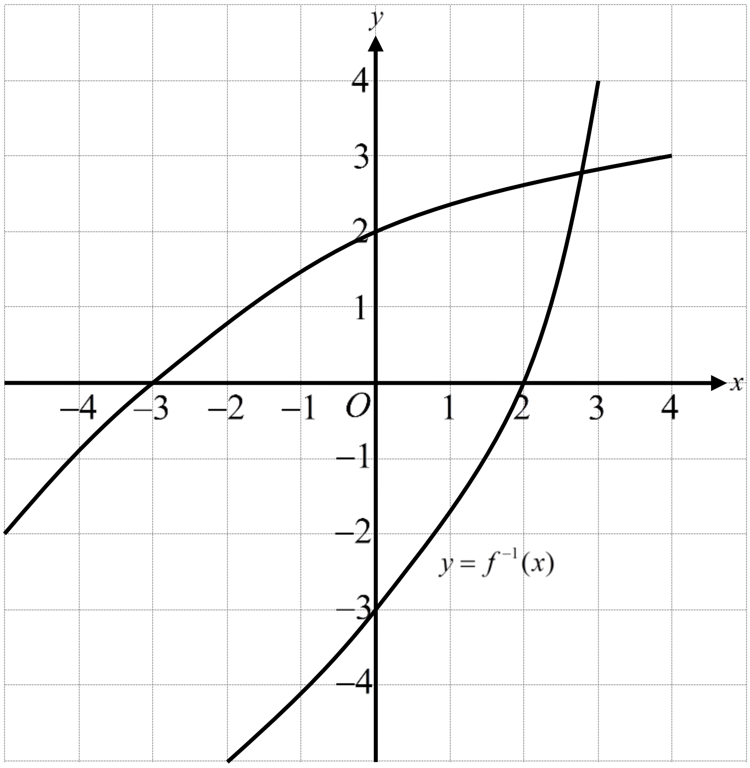
$= (0, -2)$ A1 N3

[3]

4. (a) For any two correct points from $(-2,-5)$, $(0,-3)$, $(2,0)$ or $(3,4)$ M1

For correct graph A2 N3

[3]



(b) The required coordinates for B

$= \left(\frac{-5}{2}, -2-3\right)$ M2

$= \left(-\frac{5}{2}, -5\right)$ A1 N3

[3]

Exercise 10

1.	(a)	$h=3,\ k=-1$	A2 N2	
				[2]
	(b)	$f(x)=-(x-3)^2-1$		
		c		
		$=f(0)$	(M1) for substitution	
		$=-(0-3)^2-1$		
		$=-10$	A1 N2	
				[2]
	(c)	$g(x)=-f(x-p)+q$	A1	
		$g(x)=(x-p-3)^2+q+1$		
		$p+3=1$	(M1) for translation	
		$p=-2$	A1 N2	
		$q+1=-5$	(M1) for translation	
		$q=-6$	A1 N2	
				[5]
	(d)	$-(x-3)^2-1=(x-1)^2-5$	M1	
		$-x^2+6x-9-1=x^2-2x+1-5$	(M1) for expansion	
		$2x^2-8x+6=0$		
		$2(x-1)(x-3)=0$		
		$x=1$ or $x=3$	A1	
		The y-coordinates		
		$=(1-1)^2-5$ or $(3-1)^2-5$	(M1) for substitution	
		$=-5$ or -1	A1 N3	
				[5]

2. (a) $h=1,\ k=-6$ A2 N2

[2]

(b) $f(x)=(x-1)^2-6$

c

$=f(0)$ (M1) for substitution

$=(0-1)^2-6$

$=-5$ A1 N2

[2]

(c) $g(x)=f(-(x-p))+q$ A1

$g(x)=(-(x-p)-1)^2+q-6$

$g(x)=(x-p+1)^2+q-6$

$p-1=3$ (M1) for translation

$p=4$ A1 N2

$q-6=-18$ (M1) for translation

$q=-12$ A1 N2

[5]

(d) $(x-1)^2-6=(x-3)^2-18$ M1

$x^2-2x+1-6=x^2-6x+9-18$ (M1) for expansion

$4x=-4$

$x=-1$ A1

The y-coordinate

$=(-1-1)^2-6$ (M1) for substitution

$=-2$ A1 N3

[5]

3.	(a)	$h = 1$	A1 N1	
				[1]
	(b)	$f(0) = 3$	(M1) for substitution	
		$3 = -(0-1)^2 + k$		
		$k = 4$	A1 N2	
				[2]
	(c)	$g(x) = r[f(x-p)+q]$	(M1) for transformation	
		$g(x) = -r(x-p-1)^2 + (4+q)r$	A1	
		$-p-1=0$	(M1) for translation	
		$p = -1$	A1 N2	
		$-r = -3$		
		$r = 3$	A1 N1	
		$(4+q)(3) = 3$		
		$4+q = 1$		
		$q = -3$	A1 N1	
				[6]
	(d)	$-(x-1)^2 + 4 = -3x^2 + 3$	M1	
		$-x^2 + 2x - 1 + 4 = -3x^2 + 3$		
		$2x^2 + 2x = 0$	A1	
		$2x(x+1) = 0$	(M1) for factorization	
		$x = 0$ or $x = -1$	A2	
		$y = 3$ or $y = 0$	(M1) for substitution	
		Therefore, the coordinates of the points of intersection are $(0, 3)$ and $(-1, 0)$.	A2 N4	
				[8]

4.	(a)	$f(x) = a(x+2)^2 + 2$	
		$f(0) = 6$	
		$a(0+2)^2 + 2 = 6$	(M1) for substitution
		$4a = 4$	
		$a = 1$	A1
		$f(x) = (x+2)^2 + 2$	
		$f(x) = x^2 + 4x + 6$	
		$\therefore b = 4,\ c = 6$	A2 N0
			[4]
	(b)	$g(x) = rf(x-p) + q$	(M1) for transformation
		$g(x) = r(x+2-p)^2 + 2r + q$	A1
		$2 - p = 0$	(M1) for translation
		$p = 2$	A1 N1
		$r = 5$	A1 N1
		$2r + q = -2$	
		$q = -12$	A1 N1
			[6]
	(c)	$x^2 + 4x + 6 = 5x^2 - 2$	M1
		$4x^2 - 4x - 8 = 0$	A1
		$4(x-2)(x+1) = 0$	(M1) for factorization
		$x = -1$ or $x = 2$	A2
		$y = 3$ or $y = 18$	(M1) for substitution
		Therefore, the coordinates of the points of intersection are $(-1, 3)$ and $(2, 18)$.	A2 N4
			[8]

Exercise 11

1. (a) $f(6)$

$= 2(6)^2 + 8(6) - 7$ (M1) for substitution

$= 113$ A1 N2

[2]

(b) $(g \circ f)(x)$

$= g(f(x))$ (M1) for composite function

$= 2x^2 + 8x - 7 - 17$

$= 2x^2 + 8x - 24$ A1 N2

[2]

(c) $(g \circ f)(x) = 0$

$2x^2 + 8x - 24 = 0$ (M1) for setting equation

$2(x+6)(x-2) = 0$

$x = -6$ or $x = 2$ A2 N3

[3]

2. (a) $f(-2)$

$= (-2)^2 + 2(-2) - 5$ (M1) for substitution

$= -5$ A1 N2

[2]

(b) $(f \circ g)(x)$

$= f(g(x))$ (M1) for composite function

$= (x+1)^2 + 2(x+1) - 5$

$= x^2 + 4x - 2$ A1 N2

[2]

(c) $(f \circ g)(x) = 0$

$x^2 + 4x - 2 = 0$ (M1) for setting equation

$$x = \frac{-4 \pm \sqrt{4^2 - 4(1)(-2)}}{2(1)}$$

$$x = \frac{-4 \pm \sqrt{24}}{2}$$

$x = -2 - \sqrt{6}$ or $x = -2 + \sqrt{6}$ A2 N3

[3]

3. (a) $(g \circ f)(x)$

$= g(f(x))$ — (M1) for composite function

$= 3(f(x)) - 4$

$= 3x^3 - 4$ — A1 N2

[2]

(b) $(g \circ f)(3)$

$= 3(3)^3 - 4$ — M1

$= 77$ — A1 N2

[2]

(c) $(g \circ f)(x) = 1025$

$3x^3 - 4 = 1025$ — (M1) for setting equation

$x^3 = 343$

$x = 7$ — A1 N2

[2]

4. (a) $(f \circ g)(x)$

$= f(g(x))$ — (M1) for composite function

$= 5(g(x)) + 1$

$= 5x^4 + 1$ — A1 N2

[2]

(b) $(f \circ g)(-3)$

$= 5(-3)^4 + 1$ — M1

$= 406$ — A1 N2

[2]

(c) $(f \circ g)(x) = 1281$

$5x^4 + 1 = 1281$ — (M1) for setting equation

$x^4 = 256$

$x = -4$ or $x = 4$ — A2 N2

[3]

Chapter 4 Solution

Exercise 12

1. (a) $\log_5 25$

$= \log_5 5^2$ (A1) for valid approach

$= 2$ A1 N2

[2]

(b) $\log_5 0.5 + \log_5 10$

$= \log_5 (0.5 \times 10)$ (A1) for correct formula

$= \log_5 5$

$= 1$ A1 N2

[2]

(c) $\log_5 4 - \log_5 500$

$= \log_5 \frac{4}{500}$ (A1) for correct formula

$= \log_5 \frac{1}{125}$

$= \log_5 5^{-3}$ (A1) for valid approach

$= -3$ A1 N2

[3]

2. (a) $\log_{0.5} 2$

$= \log_{0.5} 0.5^{-1}$ (A1) for valid approach

$= -1$ A1 N2

[2]

(b) $\log_{0.5} \frac{1}{7} + \log_{0.5} 7$

$= \log_{0.5} \left(\frac{1}{7} \times 7\right)$ (A1) for correct formula

$= \log_{0.5} 1 = 0$ A1 N2

[2]

(c) $\log_{0.5} 24 - \log_{0.5} 3$

$= \log_{0.5} \frac{24}{3}$ (A1) for correct formula

$= \log_{0.5} 8$

$= \log_{0.5} 0.5^{-3}$ (A1) for valid approach

$= -3$ A1 N2

[3]

3. (a) $\log_2 112 - \log_2 7$

$= \log_2 \frac{112}{7}$ (A1) for correct formula

$= \log_2 16$

$= \log_2 2^4$ (A1) for valid approach

$= 4$ A1 N2

[3]

(b) $27^{\log_3 2}$

$= 3^{3\log_3 2}$ M1(A1) for valid approach

$= 3^{\log_3 2^3}$

$= 3^{\log_3 8}$ (A1) for valid approach

$= 8$ A1 N3

[4]

4. (a) $\log_3 \frac{1}{3} + \log_3 45 - \log_3 15$

$= \log_3 \left(\frac{1}{3} \times 45 \div 15 \right)$ (A1) for correct formula

$= \log_3 1$ (A1) for correct value

$= 0$ A1 N2

[3]

(b) $25^{\log_5 7}$

$= 5^{2\log_5 7}$ M1(A1) for valid approach

$= 5^{\log_5 7^2}$

$= 5^{\log_5 49}$ (A1) for valid approach

$= 49$ A1 N3

[4]

Exercise 13

1.	(a)	$y = \log_5 \sqrt[3]{x}$			
		$\Rightarrow x = \log_5 \sqrt[3]{y}$	M1		
		$\sqrt[3]{y} = 5^x$	A1		
		$y = (5^x)^3$			
		$\therefore f^{-1}(x) = 5^{3x}$	AG	N0	
					[2]
	(b)	$\{y : y > 0\}$	A1	N1	
					[1]
	(c)	$(f^{-1} \circ g)(5)$			
		$= f^{-1}(g(5))$			
		$= f^{-1}(\log_5 25)$	(M1) for substitution		
		$= f^{-1}(2)$	(A1) for correct value		
		$= 5^{3(2)}$	(M1) for substitution		
		$= 5^6$			
		$= 15625$	A1	N2	
					[4]
2.	(a)	$y = e^{4x}$			
		$\Rightarrow x = e^{4y}$	M1		
		$4y = \ln x$	A1		
		$y = 0.25 \ln x$			
		$\therefore f^{-1}(x) = 0.25 \ln x$	AG	N0	
					[2]
	(b)	$\{x : x > 0\}$	A1	N1	
					[1]
	(c)	$(g \circ f^{-1})(16)$			
		$= g(f^{-1}(16))$			
		$= g(0.25 \ln 16)$	(M1) for substitution		
		$= g(\ln 16^{0.25})$			
		$= g(\ln 2)$	(A1) for correct value		
		$= (e^{\ln 2} - 1)^3$	(M1) for substitution		
		$= (2-1)^3$			
		$= 1$	A1	N2	
					[4]

3.	(a)	$y = \ln x + 3$			
		$\Rightarrow x = \ln y + 3$	M1		
		$x - 3 = \ln y$	A1		
		$y = e^{x-3}$			
		$\therefore f^{-1}(x) = e^{x-3}$	AG	N0	
					[2]
	(b)	$\{y : y > 0\}$	A1	N1	
					[1]
	(c)	$(f \circ g)(2)$			
		$= f(g(2))$			
		$= f(e^{(2+1)(2-3)})$	(M1) for substitution		
		$= f(e^{-3})$	(A1) for correct value		
		$= \ln e^{-3} + 3$	(M1) for substitution		
		$= -3 + 3$			
		$= 0$	A1	N2	
					[4]
4.	(a)	$y = 2^{3x}$			
		$\Rightarrow x = 2^{3y}$	M1		
		$3y = \log_2 x$	A1		
		$y = \frac{1}{3}\log_2 x$			
		$\therefore f^{-1}(x) = \frac{1}{3}\log_2 x$	AG	N0	
					[2]
	(b)	$\{y : y \in \mathbb{R}\}$	A1	N1	
					[1]
	(c)	$(g \circ f)(x)$			
		$= g(f(x))$			
		$= g(2^{3x})$	(M1) for substitution		
		$= (1 + \log_2 2^{3x})^2$	(M1) for substitution		
		$= (1 + 3x)^2$	(A1) for correct value		
		$= 9x^2 + 6x + 1$	A1	N2	
					[4]

Exercise 14

1. $\log_2 16x - \log_2(2-x) = 4$

$\log_2 \dfrac{16x}{2-x} = 4$ M1A1

$\dfrac{16x}{2-x} = 2^4$ A1

$\dfrac{16x}{2-x} = 16$

$16x = 16(2-x)$ M1

$16x = 32 - 16x$

$32x = 32$

$x = 1$ A1 N2

[5]

2. $2^{x^2} \times 2^{2(3x+4)} = 8$

$2^{x^2+2(3x+4)} = 2^3$ (M1)A1 for valid approach

$x^2 + 2(3x+4) = 3$ (M1)A1 for valid approach

$x^2 + 6x + 8 = 3$

$x^2 + 6x + 5 = 0$

$(x+1)(x+5) = 0$ (M1) for factorization

$x+1=0$ or $x+5=0$

$x=-1$ or $x=-5$ A2 N3

[7]

3. $\log_k \dfrac{8x - x^2}{4} = 2$

$\dfrac{8x - x^2}{4} = k^2$ (M1) for valid approach

$8x - x^2 = 4k^2$ (A1) for correct formula

$x^2 - 8x + 4k^2 = 0$ has exactly one solution. A1

Thus the discriminant of the equation will be zero. R1

$(-8)^2 - 4(1)(4k^2) = 0$ M1A1

$64 - 16k^2 = 0$

$k^2 = 4$

$k = -2$ (*Rejected*) or $k = 2$ A1 N3

[7]

4. $\log_3(6x - kx^2) = 1$

$6x - kx^2 = 3^1$	(M1) for valid approach
$kx^2 - 6x + 3 = 0$ has two distinct real solutions.	A1
Thus the discriminant of the about equation will be positive.	R1
$(-6)^2 - 4(k)(3) > 0$	M1A1
$36 - 12k > 0$	
$12k < 36$	
$k < 3$	(A1) for correct inequality
Therefore, the range is $0 < k < 3$.	A1 N3
	[7]

Exercise 15

1. (a) $f(x)=0$ (M1) for setting equation

$x=0.5345225$

$x=0.535$ A1 N2

[2]

(b) The maximum point is $(2.13, 3.54)$. A2 N2

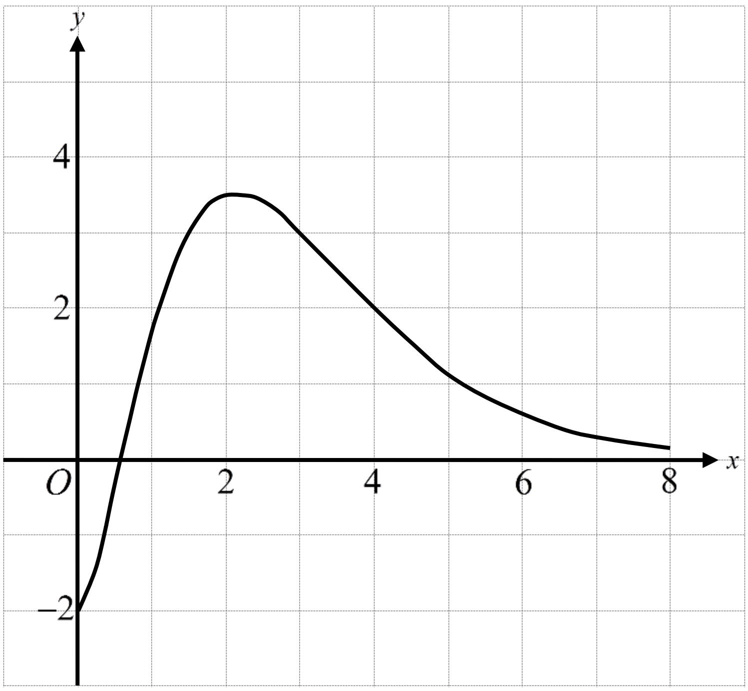
[2]

(c) For correct domain and endpoints at $x=0$ and $x=8$ A1

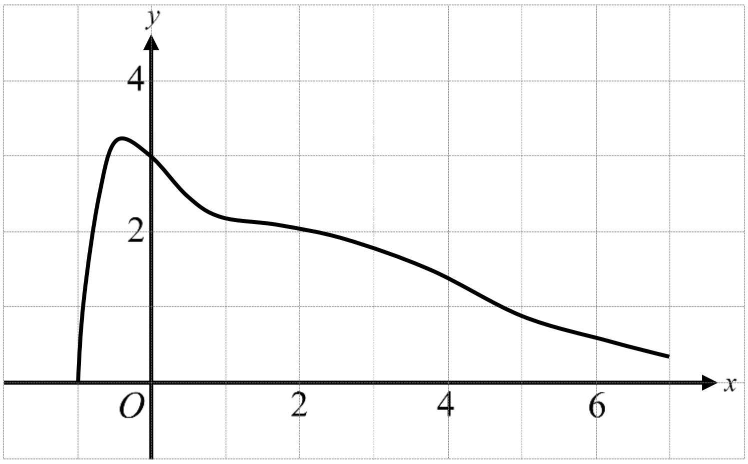
For correct maximum point A1

For correct shape A1 N3

[3]



2.	(a)	$f(x)=0$	(M1) for setting equation		
		$x=-1$			
		$x=0$			
		$y=3$			
		Thus, the x-intercept and the y-intercept are -1 and 3 respectively.	A2	N2	
					[3]
	(b)	The maximum point is $(-0.325, 3.20)$.	A2	N2	
					[2]
	(c)	For correct domain, end-points and endpoints	A1		
		For correct maximum point	A1		
		For correct shape	A1	N3	
					[3]



3. (a) $f(x) = 0$ (M1) for setting equation

$x = 3.662041$

$x = 3.66$ A1 N2

[2]

(b) $y = -3$ A2 N2

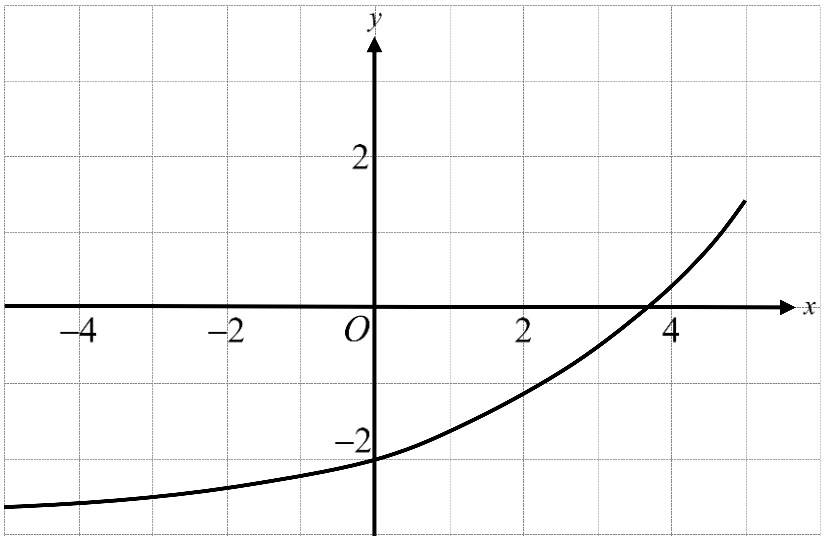
[2]

(c) For correct domain, end-points and intercept A1

For correct asymptote A1

For correct shape A1 N3

[3]



4. (a) $(2.5, 3.05)$ A2 N2

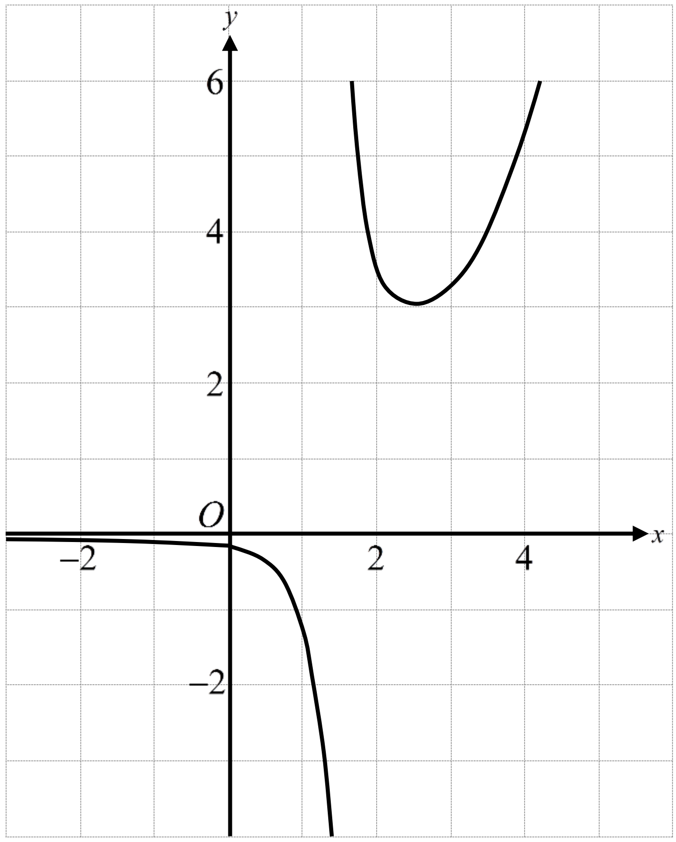
[2]

(b) $x = 1.5$ A2 N2

[2]

(c) For correct domain, end-points and intercept A1
For correct asymptote A1
For correct shape A1 N3

[3]



Exercise 16

1. (a) Initial number

$= 2500e^{0.075(0)}$ (M1) for substitution

$= 2500$ A1 N2

[2]

(b) The required number

$= 2500e^{0.075(10)}$ (M1) for substitution

$= 5292.500042$ A1

$= 5290$ A1 N2

[3]

(c) $8000 = 2500e^{0.075t}$ M1

$e^{0.075t} = 3.2$ (A1) for correct equation

$0.075t = \ln 3.2$

$t = \frac{1}{0.075}\ln 3.2$

$t = 15.5$

Thus, it takes 15.5 years to reach 8000 leopards. A1 N2

[3]

(d) $B(10) = 5000$ (M1) for setting equation

$ke^{\frac{1800}{k}} = 5000$

$ke^{\frac{1800}{k}} - 5000 = 0$

By considering the graph of $y = ke^{\frac{1800}{k}} - 5000$ (M1) for valid approach

$k = 1472.0674$

$\therefore k = 1470$ A1 N3

[3]

(e) $B(t) > A(t)$ (M1) for setting inequality

$B(t) - A(t) > 0$

$1472.0674e^{\frac{180}{1472.0674}t} - 2500e^{0.075t} > 0$

By considering the graph of

$y = 1472.0674e^{\frac{180}{1472.0674}t} - 2500e^{0.075t}$, (M1) for valid approach

$t > 11.202547$ (A1) for correct inequality

$\therefore n = 12$ A1 N3

[4]

2. (a) Initial number

$= 420 \times 1.15^0$ (M1) for substitution

$= 420$ A1 N2

[2]

(b) The required number

$= 420 \times 1.15^6$ (M1) for substitution

$= 971.4855216$

$= 971$ A1 N2

[2]

(c) $420 \times 1.15^t = 750$ A1

$420 \times 1.15^t - 750 = 0$

By considering the graph of $y = 420 \times 1.15^t - 750$ (M1) for valid approach

$t = 4.148615$

Thus, it takes 4.15 years to reach 750 trams. A1 N2

[3]

(d) $\dfrac{4680000}{70e^{-5k} + 130} = 27500$ A1

$70e^{-5k} + 130 = \dfrac{1872}{11}$

$70e^{-5k} = \dfrac{442}{11}$

$e^{-5k} = \dfrac{221}{385}$ (M1) for valid approach

$-5k = \ln \dfrac{221}{385}$

$k = 0.1110161266$

$k = 0.111$ A1 N3

[3]

(e) $420 \times 1.15^n > 5\left(\dfrac{4680000}{70e^{-0.1110161266n} + 130}\right)$ M1A1

$420 \times 1.15^n - \dfrac{23400000}{70e^{-0.1110161266n} + 130} > 0$

By considering the graph of

$y = 420 \times 1.15^n - \dfrac{23400000}{70e^{-0.1110161266n} + 130}$, (M1) for valid approach

$n > 43.331409$

$\therefore n = 44$ A1 N4

[4]

3. (a) Initial number

$=1050\times1.25^0$ (M1) for substitution

$=1050$ A1 N2

[2]

(b) The required number

$=1050\times1.25^{16}$ (M1) for substitution

$=37303.49363$

$=37300$ A1 N2

[2]

(c) $1050\times1.25^t=4200$ A1

$1.25^t=4$

$\ln1.25^t=\ln4$ (M1) for valid approach

$t\ln1.25=\ln4$

$t=\dfrac{\ln4}{\ln1.25}$

$t=6.212567439$

Thus, it takes 6.21 weeks to reach 4200 cars. A1 N2

[3]

(d) $\dfrac{410000}{95e^{-12k}+75k}=4600$ A1

$\dfrac{410000}{95e^{-12k}+75k}-4600=0$

By considering the graph of

$y=\dfrac{410000}{95e^{-12k}+75k}-4600$, (M1) for valid approach

$k=1.188405$

$k=1.19$ A1 N3

[3]

(e) $1050\times1.25^n>2\left(\dfrac{410000}{95e^{-1.188405n}+75(1.188405)}\right)$ M1A1

$1050\times1.25^n-\dfrac{820000}{95e^{-1.188405n}+75(1.188405)}>0$

By considering the graph of

$y=1050\times1.25^n-\dfrac{820000}{95e^{-1.188405n}+75(1.188405)}$, (M1) for valid approach

$n>9.7264913$

$\therefore n=10$ A1 N4

[4]

4. (a) Initial pressure

$= 4 \times e^{0.12(30)}$ (M1) for substitution

$= 146.3929378$

$= 146$ A1 N2

[2]

(b) $4e^{0.12t} = 8$ A1

$e^{0.12t} = 2$

$0.12t = \ln 2$ (M1) for substitution

$t = 5.776226505$

Hence it takes 5.78 minutes to reach 8 units. A1 N2

[3]

(c) $Q(0) = 3.5$

$Q_0 e^{k(0)} = 3.5$

$Q_0 = 3.5$ A1

$Q(30) = 171$

$3.5e^{k(30)} = 171$ (M1) for substitution

$e^{30k} = 48.85714286$

$30k = \ln 48.85714286$

$k = 0.1296300196$

$k = 0.130$ A1 N3

[3]

(d) $4e^{0.12n} + 3.5e^{0.1296300196n} > 400$ M1A1

$4e^{0.12n} + 3.5e^{0.1296300196n} - 400 > 0$

By considering the graph of

$y = 4e^{0.12n} + 3.5e^{0.1296300196n} - 400$, (M1) for valid approach

$n > 31.847494$

$\therefore n = 32$ A1 N4

[4]

Chapter 5 Solution

Exercise 17

1. (a) $d=\dfrac{u_5-u_1}{5-1}$ (M1) for finding d

$d=\dfrac{-1-27}{4}$

$d=-7$ A1 N2

[2]

(b) $u_{25}=u_1+(25-1)d$ (A1) for correct formula

$u_{25}=27+(25-1)(-7)$

$u_{25}=-141$ A1 N2

[2]

(c) $S_{25}=\dfrac{25}{2}\left[2u_1+(25-1)d\right]$ (A1) for correct formula

$S_{25}=\dfrac{25}{2}\left[2(27)+(25-1)(-7)\right]$

$S_{25}=-1425$ A1 N2

[2]

2. (a) $d=\dfrac{u_7-u_1}{7-1}$ (M1) for finding d

$d=\dfrac{6.5-3.5}{6}$

$d=0.5$ A1 N2

[2]

(b) $u_{42}=u_1+(42-1)d$ (A1) for correct formula

$u_{42}=3.5+(42-1)(0.5)$

$u_{42}=24$ A1 N2

[2]

(c) $S_{84}=\dfrac{84}{2}\left[2u_1+(84-1)d\right]$ (A1) for correct formula

$S_{84}=\dfrac{84}{2}\left[2(3.5)+(84-1)(0.5)\right]$

$S_{84}=2037$ A1 N2

[2]

3. (a) $d = \dfrac{u_{10} - u_2}{10 - 2}$ (M1) for finding d

$d = \dfrac{24 - 0}{8}$

$d = 3$ A1 N2

[2]

(b) $u_4 = u_2 + 2d$ (A1) for correct formula

$u_4 = 0 + (2)(3)$

$u_4 = 6$ A1 N2

[2]

(c) $S_{10} = \dfrac{10}{2}\left[2u_1 + (10-1)d\right]$ (A1) for correct formula

$S_{10} = \dfrac{10}{2}\left[2(-3) + (10-1)(3)\right]$

$S_{10} = 105$ A1 N2

[2]

4. (a) $d = \dfrac{u_8 - u_3}{8 - 3}$ (M1) for finding d

$d = \dfrac{-\dfrac{22}{3} - \left(-\dfrac{2}{3}\right)}{5}$

$d = -\dfrac{4}{3}$ A1 N2

[2]

(b) $u_{11} = u_8 + 3d$ (A1) for correct formula

$u_{11} = -\dfrac{22}{3} + 3\left(-\dfrac{4}{3}\right)$

$u_{11} = -\dfrac{34}{3}$ A1 N2

[2]

(c) $S_{40} = \dfrac{40}{2}\left[2u_1 + (40-1)d\right]$ (A1) for correct formula

$S_{40} = \dfrac{40}{2}\left[2(2) + (40-1)\left(-\dfrac{4}{3}\right)\right]$ (A1) for substitution

$S_{40} = -960$ A1 N3

[3]

Exercise 18

1. (a) $d = -3$ — (A1) for correct value

$u_{83} = 50 + (83-1)(-3)$ — (A1) for correct formula

$u_{83} = -196$ — A1 N3

[3]

(b) $-319 = 50 + (n-1)(-3)$ — (M1) for valid approach

$-3(n-1) = -369$ — (A1) for correct equation

$n - 1 = 123$

$n = 124$ — A1 N2

[3]

2. (a) $d = 0.5$ — (A1) for correct value

$u_n = 17.5 + (n-1)(0.5)$ — (M1) for valid approach

$u_n = 0.5n + 17$ — A1 N3

[3]

(b) $40 = 0.5n + 17$ — (M1) for substitution

$23 = 0.5n$ — (A1) for simplification

$n = 46$ — A1 N2

[3]

3. (a) $d = 5$ — A1

$251 = 6 + (n-1)(5)$ — M1

$245 = 5(n-1)$ — A1

$49 = n - 1$

$n = 50$ — AG N0

[3]

(b) $S_{50} = \frac{50}{2}\left[2u_1 + (50-1)d\right]$ — M1

$S_{50} = \frac{50}{2}\left[2(6) + (50-1)(5)\right]$ — (A1) for substitution

$S_{50} = 6425$ — A1 N2

[3]

4. (a) $d = 11$ — A1

$221 = 12 + (n-1)(11)$ — M1

$209 = 11(n-1)$ — A1

$19 = n - 1$

$n = 20$ — AG N0

[3]

(b) The total number

$= u_{18} + u_{19} + u_{20}$ — M1

$= (221-22) + (221-11) + 221$ — (A1) for substitution

$= 630$ — A1 N2

[3]

Exercise 19

1. $S_n = \frac{n}{2}[u_1 + u_n]$ (M1) for valid approach

$-5320 = \frac{80}{2}[u_1 - 185]$ A1

$-133 = u_1 - 185$

$u_1 = 52$ A1 N2

$-185 = 52 + (80-1)d$ (M1) for substitution

$79d = -237$ A1

$d = -3$ A1 N2

[6]

2. $S_n = \frac{n}{2}[u_1 + u_n]$ (M1) for valid approach

$4512 = \frac{96}{2}[u_1 + 85]$ A1

$94 = u_1 + 85$

$u_1 = 9$ A1 N2

$85 = 9 + (96-1)d$ (M1) for substitution

$76 = 95d$ A1

$d = \frac{4}{5}$ A1 N2

[6]

3. (a) $u_3 = 90$

$u_1 + (3-1)d = 90$ (M1) for substitution

$u_1 + 2d = 90$ A1

$S_{10} = 1375$

$\frac{10}{2}\left[2u_1 + (10-1)d\right] = 1375$ (M1) for substitution

$2u_1 + 9d = 275$ A1

Solving, we have $u_1 = 52$ and $d = 19$. A2 N4

[6]

(b) u_{19}

$= 52 + (19-1)(19)$ (M1) for substitution

$= 394$ A1 N2

[2]

4. (a) $u_9 = 276$

$u_1 + (9-1)d = 276$ (M1) for substitution

$u_1 + 8d = 276$ A1

$S_6 = 5880$

$\frac{6}{2}\left[2u_1 + (6-1)d\right] = 5880$ (M1) for substitution

$2u_1 + 5d = 1960$ A1

Solving, we have $u_1 = 1300$ and $d = -128$. A2 N4

[6]

(b) S_{12}

$= \frac{12}{2}\left[2(1300) + (12-1)(-128)\right]$ (M1) for substitution

$= 7152$ A1 N2

[2]

Exercise 20

1.	(a)	$d = 13 - 12.3$	(M1) for valid approach
		$d = 0.7$	A1 N2
			[2]
	(b)	$u_{50} = 12.3 + (50-1)(0.7)$	(M1) for substitution
		$u_{50} = 46.6$	A1 N2
			[2]
	(c)	$S_{50} = \frac{50}{2}\left[2(12.3) + (50-1)(0.7)\right]$	(M1) for substitution
		$S_{50} = 1472.5$	A1 N2
			[2]
2.	(a)	$d = 52 - 50$	
		$d = 2$	A1 N1
			[1]
	(b)	$u_n = u_1 + (n-1)d$	(M1) for valid approach
		$50 = u_1 + (5-1)(2)$	A1
		$u_1 = 42$	A1 N3
			[3]
	(c)	$S_{20} = \frac{20}{2}\left[2(42) + (20-1)(2)\right]$	(M1) for substitution
		$S_{20} = 1220$	A1 N2
			[2]
3.	(a)	$d = 21 - 24$	
		$d = -3$	A1 N1
			[1]
	(b)	$u_1 = 24 - 12(-3)$	
		$u_1 = 60$	A1
		$u_n = u_1 + (n-1)d$	M1
		$u_n = 60 + (n-1)(-3)$	A1
		$u_n = 60 - 3n + 3$	
		$u_n = 63 - 3n$	AG N0
			[3]
	(c)	S_3	
		$= \frac{3}{2}\left[2(60) + (3-1)(-3)\right]$	(M1) for substitution
		$= 171$	A1 N2
			[2]

4. (a) $d = \frac{11}{14} - \frac{5}{7}$ (M1) for valid approach

$d = \frac{1}{14}$ A1 N2

[2]

(b) $u_{30} = \frac{5}{7} + (30-1)\left(\frac{1}{14}\right)$ (M1) for substitution

$u_{30} = \frac{39}{14}$ A1 N2

[2]

(c) $S_n = \frac{n}{2}\left[2u_1 + (n-1)d\right]$

$S_n = \frac{n}{2}\left[2\left(\frac{5}{7}\right) + (n-1)\left(\frac{1}{14}\right)\right]$ A1

$S_n = \frac{n}{2}\left[\frac{10}{7} + \frac{1}{14}n - \frac{1}{14}\right]$ A1

$S_n = \frac{n}{2}\left(\frac{19+n}{14}\right)$

$S_n = \frac{n^2 + 19n}{28}$ AG N0

[2]

Chapter 6 Solution

Exercise 21

1. (a) $r = \frac{1}{4}$ A1 N1

[1]

(b) $u_8 = u_1 \times r^{8-1}$

$u_8 = 1024 \times \left(\frac{1}{4}\right)^{8-1}$ (A1) for substitution

$u_8 = \frac{1}{16}$ A1 N2

[2]

(c) $S_\infty = \frac{u_1}{1-r}$

$S_\infty = \frac{1024}{1-\frac{1}{4}}$ A1

$S_\infty = \frac{4096}{3}$ A1 N1

[2]

2. (a) $r = \frac{\ln x^{24}}{\ln x^{48}} = \frac{24\ln x}{48\ln x}$ M1A1

$r = \frac{1}{2}$ A1 N1

[3]

(b) $u_6 = u_1 \times r^{6-1}$

$u_6 = 48\ln x \times \left(\frac{1}{2}\right)^{6-1}$ (A1) for substitution

$u_6 = \frac{3}{2}\ln x$ A1 N2

[2]

(c) $S_\infty = \frac{u_1}{1-r}$

$S_\infty = \frac{48\ln x}{1-\frac{1}{2}}$ A1

$S_\infty = 96\ln x$ A1 N1

[2]

3. (a) $r = \dfrac{e^{8x}}{e^{12x}}$ M1

$r = e^{-4x}$ A1 N1

[2]

(b) $u_7 = u_1 \times r^{7-1}$

$u_7 = e^{12x} \times (e^{-4x})^{7-1}$ (A1) for substitution

$u_7 = e^{-12x}$ A1 N2

[2]

(c) $S_\infty = \dfrac{u_1}{1-r}$

$S_\infty = \dfrac{e^{12x}}{1-e^{-4x}}$

$S_\infty = \dfrac{e^{16x}}{e^{4x}-1}$ (A1) for correct working

$\dfrac{e^{16x}}{e^{4x}-1} = \dfrac{e^{96}}{e^{24}-1}$ M1

$x = 6$ A1 N1

[3]

4. (a) $r = \dfrac{3^{9x}}{3^{10x}}$ M1

$r = 3^{-x}$ A1 N1

[2]

(b) $u_n = u_1 \times r^{n-1}$

$u_n = 3^{10x} \times (3^{-x})^{n-1}$ (A1) for substitution

$u_n = 3^{10x} \times 3^{(1-n)x}$ (M1) for valid approach

$u_n = 3^{(11-n)x}$ A1 N2

[3]

(c) $3^{-x} = \dfrac{1}{3}$ M1

$3^{-x} = 3^{-1}$

$x = 1$

$S_\infty = \dfrac{u_1}{1-r}$

$S_\infty = \dfrac{3^{10}}{1-3^{-1}}$ (A1) for substitution

$S_\infty = \dfrac{1}{2} \times 3^{11}$ A1 N1

[3]

Exercise 22

1.	(a)	(i)	$r = \dfrac{10}{m+5}$ or $\dfrac{m-10}{10}$	A1	N1	
		(ii)	$\dfrac{10}{m+5} = \dfrac{m-10}{10}$	A1		
			$(m-10)(m+5) = 100$	A1		
			$m^2 - 5m - 50 = 100$	M1		
			$m^2 - 5m - 150 = 0$	AG	N0	
						[4]
	(b)	(i)	$m^2 - 5m - 150 = 0$			
			$(m-15)(m+10) = 0$	M1		
			$m = 15$ or $m = -10$	A2	N3	
		(ii)	Case 1: $m = 15$			
			$r = \dfrac{10}{15+5}$	(M1) for substitution		
			$r = \dfrac{1}{2}$	A1		
			Case 2: $m = -10$			
			$r = \dfrac{10}{-10+5}$			
			$r = -2$	A1	N3	
						[6]
	(c)	(i)	$r = \dfrac{1}{2}$ leads to a finite sum.	A1		
			As $-1 < \dfrac{1}{2} < 1$.	R1	N0	
		(ii)	$u_1 = 15 + 5$	(A1) for substitution		
			$u_1 = 20$	(A1) for correct value		
			$S_\infty = \dfrac{u_1}{1-r}$			
			$S_\infty = \dfrac{20}{1-\dfrac{1}{2}}$	A1		
			$S_\infty = 40$	A1	N3	
						[6]

2.	(a)	(i)	$r=\frac{9}{m-12}$ or $\frac{m+12}{9}$	A1	N1	
		(ii)	$\frac{9}{m-12}=\frac{m+12}{9}$	A1		
			$(m-12)(m+12)=81$	A1		
			$m^2-144=81$	M1		
			$m^2-225=0$	AG	N0	
						[4]
	(b)	(i)	$m^2-225=0$			
			$(m-15)(m+15)=0$	M1		
			$m=15$ or $m=-15$	A2	N3	
		(ii)	Case 1: $m=15$			
			$r=\frac{9}{15-12}$	(M1) for substitution		
			$r=3$	A1		
			Case 2: $m=-15$			
			$r=\frac{9}{-15-12}$			
			$r=-\frac{1}{3}$	A1	N3	
						[6]
	(c)	(i)	$r=-\frac{1}{3}$ leads to a finite sum.	A1		
			As $-1<-\frac{1}{3}<1$.	R1	N0	
		(ii)	$r=3$	(A1) for correct value		
			$u_1=3$	(A1) for correct value		
			$S_4=\frac{u_1(1-r^4)}{1-r}$			
			$S_4=\frac{3(1-3^4)}{1-3}$	A1		
			$S_4=120$	A1	N3	
						[6]

3. (a) (i) $r = \frac{2}{m+1}$ or $\frac{m-2}{2}$ A1 N1

(ii) $\frac{2}{m+1} = \frac{m-2}{2}$ A1

$(m+1)(m-2) = 4$ A1

$m^2 - m - 2 = 4$ M1

$m^2 - m - 6 = 0$ AG N0

[4]

(b) (i) $m^2 - m - 6 = 0$

$(m-3)(m+2) = 0$ M1

$m = 3$ or $m = -2$ A2 N3

(ii) Case 1: $m = 3$

$r = \frac{2}{3+1}$ (M1) for substitution

$r = \frac{1}{2}$ A1

Case 2: $m = -2$

$r = \frac{2}{-2+1}$

$r = -2$ A1 N3

[6]

(c) $m = 3$ leads to a finite sum.

$u_1 = \log_2 x^4$ A1

$u_1 = 4\log_2 x$

$S_\infty = \frac{4\log_2 x}{1-\frac{1}{2}}$

$S_\infty = 8\log_2 x$ A1 N2

[2]

(d) $u_1 = \log_2 \left(\frac{1}{2}\right)^{-2+1}$ A1

$u_1 = 1$ A1

$S_6 = \frac{1-\left(\frac{1}{2}\right)^6}{1-\frac{1}{2}}$ M1

$S_6 = \frac{63}{32}$ A1 N2

[4]

4. (a) (i) $r=\dfrac{9}{m+2}$ or $\dfrac{m+17}{36}$ A1 N1

(ii) $\dfrac{9}{m+2}=\dfrac{m+17}{36}$ A1

$(m+2)(m+17)=324$ A1

$m^2+19m-290=0$ M1

$(m-10)(m+29)=0$ M1

$m=10$ or $m=-29$ AG N0

[5]

(b) Case 1: $m=10$

$r=\dfrac{9}{10+2}$ (M1) for substitution

$r=\dfrac{3}{4}$ A1

Case 2: $m=-29$

$r=\dfrac{9}{-29+2}$

$r=-\dfrac{1}{3}$ A1 N2

[3]

(c) (i) $r=\dfrac{3}{4}$ leads to a finite sum. A1

As r must be positive and $-1<\dfrac{3}{4}<1$. R1 N0

(ii) Recognize that the areas also form a geometric sequence M1

$a_1=\left(12\div\dfrac{3}{4}\right)^2$

$a_1=256$ (A1) for correct value

$r=\left(\dfrac{3}{4}\right)^2$

$r=\dfrac{9}{16}$ (A1) for correct value

$S_\infty=\dfrac{256}{1-\dfrac{9}{16}}$ M1

$S_\infty=\dfrac{4096}{7}$ A1 N3

[7]

Exercise 23

1. (a) $r=\frac{720}{800}$ (M1) for valid approach

$r=0.9$ A1 N2

[2]

(b) $S_6=\frac{u_1(1-r^6)}{1-r}$

$S_6=\frac{800(1-(0.9)^6)}{1-0.9}$ (A1) for substitution

$S_6=3748.472$

$S_6=3750$ A1 N2

[2]

(c) $S_n>6400$

$\frac{800(1-(0.9)^n)}{1-0.9}>6400$ (M1) for substitution

$800(1-(0.9)^n)>640$

$1-(0.9)^n>0.8$

$0.2-(0.9)^n>0$ (A1) for correct inequality

By considering the graph of $y=0.2-(0.9)^n$,

$n>15.275532$

Thus, the least value of n is 16. A1 N1

[3]

2. (a) $r = \dfrac{768}{576}$ (M1) for valid approach

$r = \dfrac{4}{3}$ A1 N2

[2]

(b) $S_7 = \dfrac{u_1(1-r^7)}{1-r}$

$S_7 = \dfrac{576\left(1-\left(\frac{4}{3}\right)^7\right)}{1-\frac{4}{3}}$ (A1) for substitution

$S_7 = 11217.38272$

$S_7 = 11200$ A1 N2

[2]

(c) $S_n < 550000$

$\dfrac{576\left(1-\left(\frac{4}{3}\right)^n\right)}{1-\frac{4}{3}} < 550000$ (M1) for substitution

$-1728\left(1-\left(\frac{4}{3}\right)^n\right) < 550000$

$-1728\left(1-\left(\frac{4}{3}\right)^n\right) - 550000 < 0$ (A1) for correct inequality

By considering the graph of

$y = -1728\left(1-\left(\frac{4}{3}\right)^n\right) - 550000$,

$n < 20.043274$

Thus, the greatest value of n is 20. A1 N1

[3]

3. (a) $r = \frac{-540}{-324}$ (M1) for valid approach

$r = \frac{5}{3}$ A1 N2

[2]

(b) $S_{10} = \frac{u_1(1-r^{10})}{1-r}$

$S_{10} = \frac{-324\left(1-\left(\frac{5}{3}\right)^{10}\right)}{1-\frac{5}{3}}$ (A1) for substitution

$S_{10} = -79889.5144$

$S_{10} = -79900$ A1 N2

[2]

(c) $S_n < -700000$

$\frac{-324\left(1-\left(\frac{5}{3}\right)^{n}\right)}{1-\frac{5}{3}} < -700000$ (M1) for substitution

$486\left(1-\left(\frac{5}{3}\right)^{n}\right) < -700000$

$486\left(1-\left(\frac{5}{3}\right)^{n}\right) + 700000 < 0$ (A1) for correct inequality

By considering the graph of

$y = 486\left(1-\left(\frac{5}{3}\right)^{n}\right) + 700000$,

$n > 14.238364$ (M1) for valid approach

Thus, the least value of n is 15. A1 N1

[4]

4. (a) $r=\frac{-2.4}{-1.5}$ (M1) for valid approach

$r=\frac{8}{5}$ A1 N2

[2]

(b) $S_n=\frac{u_1(1-r^n)}{1-r}$

$S_n=\frac{-1.5\left(1-\left(\frac{8}{5}\right)^n\right)}{1-\frac{8}{5}}$ A1

$S_n=\frac{-5\left(1-\left(\frac{8}{5}\right)^n\right)}{2}$ A1

$S_n=\frac{5^{1-n}}{2}(5^n-8^n)$ AG N0

[2]

(c) $S_n>-100$

$\frac{5^{1-n}}{2}(5^n-8^n)>-100$ (M1) for setting inequality

$\frac{5^{1-n}}{2}(5^n-8^n)+100>0$ (A1) for correct inequality

By considering the graph of

$y=\frac{5^{1-n}}{2}(5^n-8^n)+100$,

$n<7.9011562$ (M1) for valid approach

Thus, the greatest value of n is 7. A1 N1

[4]

Exercise 24

1. (a) R

$= \left(1+\frac{3.7}{(4)(100)}\right)^4$ (M1)(A1) for correct formula

$= 1.037516548$

$= 1.0375$ A1 N3

[3]

(b) $3P = P \times 1.037516548^n$ (M1)(A1) for correct formula

$3 = 1.037516548^n$

$1.037516548^n - 3 = 0$

$n = 29.82934$

Thus, the amount of money in Zoe's account will become three times the amount she invested in 2059. A1 N3

[3]

2. (a) R

$= \left(1+\frac{5.1}{(12)(100)}\right)^{12}$ (M1)(A1) for correct formula

$= 1.052209176$

$= 1.0522$ A1 N3

[3]

(b) $400000 = 180000 \times 1.052209176^n$ (M1)(A1) for correct formula

$\frac{20}{9} = 1.052209176^n$

$1.052209176^n - \frac{20}{9} = 0$

$n = 15.690261$

Thus, the amount of money in Jane's account will become \$400000 in 2037. A1 N3

[3]

3. (a) $2300 = P\left(1+\dfrac{2.9}{(4)(100)}\right)^{(4)(7)}$ (M1)(A1) for correct formula

$2300 = 1.22417563P$

$P = 1878.815379$

$P = 1878.815$ A1 N3

[3]

(b) Let R be the rate of depreciation.

$2300(1-R)^5 = 200$ (M1)(A1) for correct formula

$(1-R)^5 = \dfrac{2}{23}$

$(1-R)^5 = \dfrac{2}{23}$

$1-R = 0.6135647938$

$R = 0.3864352062$

Thus, the rate at which the cup depreciated per year is 38.6%. A1 N3

[3]

4. (a) $290000 = P\left(1+\dfrac{7.3}{(12)(100)}\right)^{(12)(9)}$ (M1)(A1) for correct formula

$290000 = 1.925161177P$

$P = 150636.7381$

$P = 150637$ A1 N3

[3]

(b) $290000(1-6.25\%)^t = 205000$ (M1)(A1) for correct formula

$0.9375^t = \dfrac{41}{58}$

$0.9375^t - \dfrac{41}{58} = 0$

$t = 5.3746342$

$t = 5.37$ A1 N3

[3]

Chapter 7 Solution

Exercise 25

1. $(2x+1)^n$

$= (1+2x)^n$

$= 1^n + \binom{n}{1}1^{n-1}(2x) + \binom{n}{2}1^{n-2}(2x)^2 + \cdots$ (M1) for valid expansion

The term in $x^2 = \binom{n}{2}(2x)^2$ (M1) for valid approach

$\binom{n}{2}2^2 = 540n$ A1

$\frac{n(n-1)}{2}(4) = 540n$ (A1) for correct working

$2(n-1) = 540$ (A1) for correct equation

$n-1 = 270$ (A1) for simplification

$n = 271$ A1 N2

[7]

2. $(3x-1)^{2n}$

$= (-1+3x)^{2n}$

$= (-1)^{2n} + \binom{2n}{1}(-1)^{2n-1}(3x) + \binom{2n}{2}(-1)^{2n-2}(3x)^2 + \cdots$ (M1) for valid expansion

The term in $x^2 = \binom{2n}{2}(-1)^{2n-2}(3x)^2$ (M1) for valid approach

$\binom{2n}{2}(-1)^{2n-2}(3)^2 = 18(2n-1)$ A1

$\frac{2n(2n-1)}{2}(9) = 18(2n-1)$ (A1) for correct working

$\frac{2n}{2}(9) = 18$ (A1) for correct equation

$9n = 18$ (A1) for simplification

$n = 2$ A1 N2

[7]

3. $(1+x)^n$

$= 1 + \binom{n}{1} x^1 + \cdots$ (M1) for valid expansion

$= 1 + nx + \cdots$ (A1) for correct values

$(1+x)^n (2+nx)$

$= (1+nx+\cdots)(2+nx)$

The coefficient of the term in x

$= (1)(n) + (n)(2)$ A2

$= 3n$

$\therefore 3n = 15$ A1

$n = 5$ A1 N1

[6]

4. $(1-x)^n$

$= 1 + \binom{n}{1} (-x)^1 + \cdots$ (M1) for valid expansion

$= 1 - nx + \cdots$ (A1) for correct values

$(1-x)^n (1-nx)^2$

$= (1-x)^n (1 - 2nx + n^2x^2)$ (A1) for expansion

$= (1-nx+\cdots)(1-2nx+n^2x^2)$

The coefficient of the term in x

$= (1)(-2n) + (-n)(1)$ A1

$= -3n$

$\therefore -3n = -99$ A1

$n = 33$ A1 N1

[6]

Exercise 26

1. (a) 17 terms — A1 N1

[1]

(b) The general term

$= \binom{16}{r}(x^3)^r\left(\frac{2}{x}\right)^{16-r}$ — (M1) for valid expansion

$= \binom{16}{r}(2)^{16-r}x^{4r-16}$

$4r-16=20$ — (M1) for finding r

$4r=36$

$r=9$ — (A1) for correct value

The required coefficient

$= \binom{16}{9}(2)^{16-9}$ — (A1) for correct working

$=1464320$ — A1 N3

[5]

2. (a) 10 terms — A1 N1

[1]

(b) The general term

$= \binom{9}{r}(5x)^r(4)^{9-r}$ — (M1) for valid expansion

$= \binom{9}{r}4^{9-r}5^r x^r$ — (M1) for valid approach

$r=3$ — (A1) for correct value

The required coefficient

$= \binom{9}{3}4^{9-3}5^3$ — (A1) for correct working

$=43008000$ — A1 N3

[5]

3. (a) 16 terms — A1 N1

[1]

(b) The general term

$=\binom{15}{r}(3x^2)^r(-8)^{15-r}$ — (M1) for valid expansion

$=\binom{15}{r}(-8)^{15-r}3^r x^{2r}$

$2r=16$ — (M1) for finding r

$r=8$ — (A1) for correct value

The required coefficient

$=\binom{15}{8}(-8)^{15-8}3^8$ — (A1) for correct working

$=-8.85\times10^{13}$ — A1 N3

[5]

4. (a) 15 terms — A1 N1

[1]

(b) The general term

$=\binom{14}{r}(2x^2)^r\left(\frac{-1}{x^2}\right)^{14-r}$ — (M1) for valid expansion

$=\binom{14}{r}(-1)^{14-r}(2)^r x^{4r-28}$

$4r-28=-8$ — (M1) for finding r

$4r=20$

$r=5$ — (A1) for correct value

The required coefficient

$=\binom{14}{5}(-1)^{14-5}(2)^5$ — (A1) for correct working

$=-64064$ — A1 N3

[5]

Exercise 27

1. The general term

$= \binom{6}{r}(kx^3)^r\left(\frac{2}{x}\right)^{6-r}$ (M1) for valid expansion

$= \binom{6}{r}(2)^{6-r}k^r x^{4r-6}$

$4r-6=2$

$4r=8$

$r=2$ (A1) for correct value

The required term

$= \binom{6}{2}(2)^{6-2}k^2 x^{4(2)-6+2}$

$=240k^2x^4$ (A1) for correct term

$240k^2x^4=6000x^4$ (M1) for setting equation

$240k^2=6000$ (A1) for correct equation

$k^2=25$

$k=5$ or $k=-5$ A1 N3

[6]

2. The general term

$= \binom{10}{r}(3x)^r\left(\frac{k}{x}\right)^{10-r}$ (M1) for valid expansion

$= \binom{10}{r}k^{10-r}3^r x^{2r-10}$

$2r-10=0$

$2r=10$

$r=5$ (A1) for correct value

The required term

$= \binom{10}{5}k^{10-5}3^5 x^{2(5)-10}(kx^2)$

$=61236k^6x^2$ (A1) for correct term

$61236k^6x^2=3919104x^2$ (M1) for setting equation

$k^6=64$ (A1) for correct equation

$k=2$ or $k=-2$ A1 N3

[6]

3. $r=4$ (A1) for correct value

The required term

$=\binom{12}{4}(x)^{12-4}(2k)^4$

$=7920k^4x^8$ (A1) for correct term

$7920k^4x^8=7920x^8$ (M1) for setting equation

$7920k^4=7920$ (A1) for correct equation

$k^4=1$

$k=1$ or $k=-1$ A1 N3

[5]

4. The general term

$=\binom{18}{r}\left(\frac{x}{k^2}\right)^r\left(\frac{k}{x}\right)^{18-r}$ (M1) for valid expansion

$=\binom{18}{r}k^{18-3r}x^{2r-18}$

$2r-18=0$

$2r=18$

$r=9$ (A1) for correct value

The required term

$=\binom{18}{9}k^{18-3(9)}x^{2(9)-18}$

$=48620k^{-9}$ (A1) for correct term

$48620k^{-9}=\frac{12155}{128}$ (M1) for setting equation

$k^{-9}=\frac{1}{512}$

$k^9=512$ (A1) for correct equation

$k=2$ A1 N3

[6]

Chapter 8 Solution

Exercise 28

1. (a) R.H.S.

$= \frac{26}{26} + \frac{1}{26}$ M1

$= \frac{27}{26}$ A1

$=$ L.H.S.

$\therefore 1 + \frac{1}{26} = \frac{27}{26}$ AG N0

[2]

(b) R.H.S.

$= 1 + \frac{1}{(m-1)(m^2+m+1)}$

$= \frac{(m-1)(m^2+m+1)}{(m-1)(m^2+m+1)} + \frac{1}{(m-1)(m^2+m+1)}$ M1

$= \frac{(m-1)(m^2+m+1)+1}{(m-1)(m^2+m+1)}$

$= \frac{m^3-1+1}{(m-1)(m^2+m+1)}$ M1A1

$= \frac{m^3}{(m-1)(m^2+m+1)}$

$=$ L.H.S.

$\therefore 1 + \frac{1}{(m-1)(m^2+m+1)} \equiv \frac{m^3}{(m-1)(m^2+m+1)}$

for $m \neq 1$ AG N0

[3]

2. (a) R.H.S.

$= \frac{1\times 9}{7\times 9} - \frac{2}{63}$ M1

$= \frac{9-2}{63}$ A1

$= \frac{7}{63}$

$= \frac{1}{9}$

$=$ L.H.S.

$\therefore \frac{1}{7} - \frac{2}{63} = \frac{1}{9}$ AG N0

[2]

(b) R.H.S.

$= \frac{1}{m-2} - \frac{2}{m(m-2)}$ M1

$= \frac{1\times m}{(m-2)\times m} - \frac{2}{m(m-2)}$

$= \frac{m-2}{m(m-2)}$ M1A1

$= \frac{1}{m}$

$=$ L.H.S.

$\therefore \frac{1}{m-2} - \frac{2}{m(m-2)} \equiv \frac{1}{m}$ for $m > 2$ AG N0

[3]

3. (a) R.H.S.

$$= \frac{2\times2\times7}{3\times2\times7} + \frac{3\times3\times7}{2\times3\times7} - \frac{4\times2\times3}{7\times2\times3}$$ M1

$$= \frac{28+63-24}{42}$$ A1

$$= \frac{67}{42}$$

$= \text{L.H.S.}$

$$\therefore \frac{2}{3} + \frac{3}{2} - \frac{4}{7} = \frac{67}{42}$$ AG N0

[2]

(b) R.H.S.

$$= \frac{2\times(m-1)\times(2m+1)}{m\times(m-1)\times(2m+1)} + \frac{3\times m\times(2m+1)}{(m-1)\times m\times(2m+1)}$$

$$- \frac{4\times m\times(m-1)}{(2m+1)\times m\times(m-1)}$$ M1

$$= \frac{(4m^2-2m-2)+(6m^2+3m)-(4m^2-4m)}{(m-1)\times m\times(2m+1)}$$ M1A1

$$= \frac{6m^2+5m-2}{m(m-1)(2m+1)}$$

$= \text{L.H.S.}$

$$\therefore \frac{6m^2+5m-2}{m(m-1)(2m+1)} \equiv \frac{2}{m} + \frac{3}{m-1} - \frac{4}{2m+1} \text{ for }$$

$m > 1$ AG N0

[3]

4. (a) R.H.S.

$$= \frac{2\times 25}{1\times 25} - \frac{4\times 5}{5\times 5} + \frac{1}{25}$$ M1

$$= \frac{50-20+1}{25}$$ A1

$$= \frac{31}{25}$$

$$= \text{L.H.S.}$$

$$\therefore \frac{31}{25} = 2 - \frac{4}{5} + \frac{1}{25}$$ AG N0

[2]

(b) R.H.S.

$$= \frac{2\times (m+1)^2}{1\times (m+1)^2} - \frac{4\times (m+1)}{(m+1)\times (m+1)} + \frac{1}{(m+1)^2}$$ M1

$$= \frac{(2m^2+4m+2)-(4m+4)+1}{(m+1)^2}$$ M1A1

$$= \frac{2m^2-1}{(m+1)^2}$$

$$= \text{L.H.S.}$$

$$\therefore \frac{2m^2-1}{(m+1)^2} = 2 - \frac{4}{(m+1)} + \frac{1}{(m+1)^2} \text{ for } m \neq -1$$ AG N0

[3]

Exercise 29

1. (a) L.H.S.

$= (3n)^2 + (3n+3)^2$

$= 9n^2 + 9n^2 + 18n + 9$ M1A1

$= 18n^2 + 18n + 9$

$=$ R.H.S. AG N0

[2]

(b) $3n$ and $3n+3$ are multiples of 3. R1

$(3n)^2 + (3n+3)^2 = 18n^2 + 18n + 9$ A1

Also $18n^2 + 18n + 9$ is an odd integer. R1

Thus, the sum of the squares of any two multiples of 3 is odd. AG N0

[3]

2. (a) L.H.S.

$= \dfrac{2n+1}{2n-1}$

$= \dfrac{2n-1+2}{2n-1}$ M1

$= \dfrac{2n-1}{2n-1} + \dfrac{2}{2n-1}$ A1

$= 1 + \dfrac{2}{2n-1}$

$=$ R.H.S. AG N0

[2]

(b) $2n+1$ and $2n+3$ are two consecutive odd integers. R1

$\dfrac{2n+1}{2n-1} = 1 + \dfrac{2}{2n-1}$ A1

Also $\dfrac{2}{2n-1}$ is a non-zero number. R1

Thus, the ratio of any odd integer to its consecutive smaller odd integer is not equal to 1. AG N0

[3]

3. (a) L.H.S.

$=(2n+1)^2(2n+3)^2$

$=(4n^2+4n+1)(4n^2+12n+9)$ M1

$=16n^4+48n^3+36n^2+16n^3+48n^2$
$+36n+4n^2+12n+9$ M1A1

$=16n^4+64n^3+88n^2+48n+9$

$=$R.H.S. AG N0

[3]

(b) $2n+1$ and $2n+3$ are two consecutive odd integers. R1

$(2n+1)^2(2n+3)^2=16n^4+64n^3+88n^2+48n+9$ A1

Also $16n^4+64n^3+88n^2+48n+9$ is an odd integer. R1

Thus, the product of the squares of any two consecutive odd integers is odd. AG N0

[3]

4. (a) L.H.S.

$=n^2+(n+1)^2+(n+2)^2$

$=n^2+n^2+2n+1+n^2+4n+4$ M1A1

$=3n^2+6n+5$

$=3n^2+6n+6-1$ M1

$=3(n^2+2n+2)-1$

$=$R.H.S. AG N0

[3]

(b) n, $n+1$ and $n+2$ are three consecutive integers. R1

$n^2+(n+1)^2+(n+2)^2=3(n^2+2n+2)-1$ A1

Also $3(n^2+2n+2)$ is a multiple of 3. R1

Thus, the sum of the squares of any three consecutive integers is smaller than a multiple of 3 by 1. AG N0

[3]

Chapter 9 Solution

Exercise 30

1. (a) The gradient of L

$= \frac{11-6}{20-10}$ (M1) for valid approach

$= \frac{1}{2}$

The equation of L:

$y-11=\frac{1}{2}(x-20)$ A1

$2y-22=x-20$

$x-2y+2=0$ A1 N2

[3]

(b) The x-intercept of L is -2 A1

The y-intercept of L is 1 A1 N2

[2]

2. (a) The gradient of L

$= \frac{-26-(-8)}{2-(-4)}$ (M1) for valid approach

$=-3$

The equation of L:

$y+8=-3(x+4)$ A1

$y+8=-3x-12$

$3x+y+20=0$ A1 N2

[3]

(b) The x-intercept of L is $-\frac{20}{3}$ A1

The y-intercept of L is -20 A1 N2

[2]

3. (a) The gradient of L_1

$= \frac{37-1}{17-5}$ (M1) for valid approach

$=3$

The equation of L_1:

$y-1=3(x-5)$ A1

$y-1=3x-15$

$3x-y-14=0$ A1 N2

[3]

(b) The gradient of L_2

$=-\frac{3}{1}$ A1

$=-3$

$\neq 3$ R1

Therefore, they are not parallel. AG N0

[2]

4. (a) The gradient of L_1

$= \frac{40-0}{4-(-4)}$ (M1) for valid approach

$=5$

The equation of L_1:

$y-0=5(x+4)$ A1

$y=5x+20$

$5x-y+20=0$ A1 N2

[3]

(b) The gradient of L_2

$=-\frac{1}{5}$ A1

The product of slopes

$=5\times-\frac{1}{5}$

$=-1$ R1

Therefore, they are perpendicular. AG N0

[2]

Exercise 31

1. (a) The gradient of L_1 is $\frac{1}{2}$ A1

The y-intercept of L_1 is 8 A1 N2

[2]

(b) The gradient of L_2 is $\frac{1}{2}$ (A1) for correct value

The equation of L_2:

$y-5=\frac{1}{2}(x+2)$ A1

$2y-10=x+2$

$x-2y+12=0$ A1 N2

[3]

2. (a) The gradient of L_1 is $-\frac{3}{2}$ A1

The x-intercept of L_1 is $\frac{4}{3}$ A1 N2

[2]

(b) The gradient of L_2 is $-\frac{3}{2}$ (A1) for correct value

The equation of L_2:

$y+7=-\frac{3}{2}(x-1)$ A1

$2y+14=3-3x$

$3x+2y+11=0$ A1 N2

[3]

3. (a) The gradient of L_1 is -3 A1

The x-intercept of L_1 is -7 A1 N2

[2]

(b) The gradient of L_2 is $\frac{1}{3}$ (A1) for correct value

The equation of L_2:

$y-0=\frac{1}{3}(x+7)$ A1

$3y=x+7$

$x-3y+7=0$ A1 N2

[3]

4. (a) The gradient of L_1 is $\frac{1}{2}$ A1

The y-intercept of L_1 is $-\frac{17}{4}$ A1 N2

[2]

(b) The gradient of L_2 is -2 (A1) for correct value

The equation of L_2:

$y+\frac{17}{4}=-2(x-0)$ A1

$4y+17=-8x$

$8x+4y+17=0$ A1 N2

[3]

Chapter 10 Solution

Exercise 32

1. (a)

$\cos\theta$

$= \sqrt{1-\sin^2\theta}$ (M1) for valid approach

$= \sqrt{1-\left(\frac{2}{3}\right)^2}$ (A1) for substitution

$= \frac{\sqrt{5}}{3}$ A1 N2

[3]

(b)

$\sin 2\theta$

$= 2\sin\theta\cos\theta$ (A1) for correct identity

$= 2\left(\frac{2}{3}\right)\left(\frac{\sqrt{5}}{3}\right)$

$= \frac{4\sqrt{5}}{9}$ A1 N2

[2]

2. (a)

$\cos\theta$

$= \frac{5}{\sqrt{(\sqrt{11})^2+5^2}}$ (M1) for valid approach

$= \frac{5}{\sqrt{36}}$ (A1) for correct value

$= \frac{5}{6}$ A1 N2

[3]

(b)

$\cos 2\theta$

$= 2\cos^2\theta - 1$ (A1) for correct identity

$= 2\left(\frac{5}{6}\right)^2 - 1$

$= \frac{7}{18}$ A1 N2

[2]

3. (a) $\cos\theta$

$= -\sqrt{1-\sin^2\theta}$ (M2) for valid approach

$= -\sqrt{1-\left(\frac{4}{5}\right)^2}$ (A1) for substitution

$= -\frac{3}{5}$ A1 N3

[4]

(b) $\sin 2\theta$

$= 2\sin\theta\cos\theta$

$= 2\left(\frac{4}{5}\right)\left(-\frac{3}{5}\right)$ (A1) for substitution

$= -\frac{24}{25}$ A1 N2

[2]

4. (a) $\tan\theta$

$= \frac{\sin\theta}{\cos\theta}$ (M1) for valid approach

$= \frac{\sqrt{1-\cos^2\theta}}{\cos\theta}$ (M1) for valid approach

$= \frac{\sqrt{1-m^2}}{m}$ A1 N3

[3]

(b) $\tan 2\theta$

$= \frac{2\tan\theta}{1-\tan^2\theta}$ (M1) for valid approach

$= \frac{2\left(\frac{\sqrt{1-m^2}}{m}\right)}{1-\left(\frac{\sqrt{1-m^2}}{m}\right)^2}$ (A1) for substitution

$= \frac{\frac{2\sqrt{1-m^2}}{m}}{1-\frac{1-m^2}{m^2}}$

$= \frac{2m\sqrt{1-m^2}}{m^2-1+m^2}$ (M1) for valid approach

$= \frac{2m\sqrt{1-m^2}}{2m^2-1}$ A1 N2

[4]

Exercise 33

1. (a) p

$= \frac{2-(-6)}{2}$ (M1) for valid approach

$= 4$ A1 N2

[2]

(b) The period of the graph is π.

q

$= \frac{2\pi}{\pi}$ (M1) for valid approach

$= 2$ A1 N2

[2]

(c) r

$= \frac{2+(-6)}{2}$ (M1) for valid approach

$= -2$ A1 N2

[2]

2. (a) p

$= \frac{60-28}{2}$ (M1) for valid approach

$= 16$ A1 N2

[2]

(b) The period of the graph is 8π.

q

$= \frac{2\pi}{8\pi}$ (M1) for valid approach

$= \frac{1}{4}$ A1 N2

[2]

(c) r

$= \frac{28+60}{2}$ (M1) for valid approach

$= 44$ A1 N2

[2]

3. (a) p

$=\dfrac{2\pi-(-2\pi)}{2}$ (M1) for valid approach

$=2\pi$ A1 N2

[2]

(b) The period of the graph is $\dfrac{\pi}{3}$.

q

$=\dfrac{2\pi}{\frac{\pi}{3}}$ (M1) for valid approach

$=6$ A1 N2

[2]

(c) $2\pi=2\pi\cos\left(6\left(\dfrac{\pi}{12}-r\right)\right)$ (M1) for setting equation

$1=\cos\left(6\left(\dfrac{\pi}{12}-r\right)\right)$

$6\left(\dfrac{\pi}{12}-r\right)=0$

$r=\dfrac{\pi}{12}$ A1 N2

[2]

4. (a) p

$=\dfrac{20-0}{2}$ (M1) for valid approach

$=10$ A1 N2

[2]

(b) The period of the graph is 6π.

q

$=\dfrac{2\pi}{6\pi}$ (M1) for valid approach

$=\dfrac{1}{3}$ A1 N2

[2]

(c) $20=10\cos\left(\dfrac{1}{3}(6\pi-r)\right)+10$ (M1) for setting equation

$\cos\left(\dfrac{1}{3}(6\pi-r)\right)=1$

$\dfrac{1}{3}(6\pi-r)=0$

$r=0$ A1 N2

[2]

Exercise 34

1. (a) (i) The amplitude of f is 3.5. A1 N1

(ii) The period of f

$=2\pi \div \pi$ (M1) for valid approach

$=2$ A1 N2

[3]

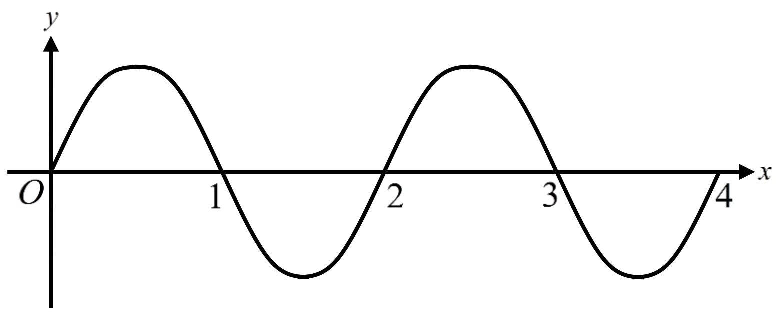
(b) For correct x-intercepts A1

For correct maximum and minimum points A1

For correct domain A1

For sinusoidal curve starting at the origin with correct period A1 N4

[4]



2. (a) (i) The amplitude of f is 3. A1 N1

(ii) The period of f

$=2\pi \div \pi$ (M1) for valid approach

$=2$ A1 N2

[3]

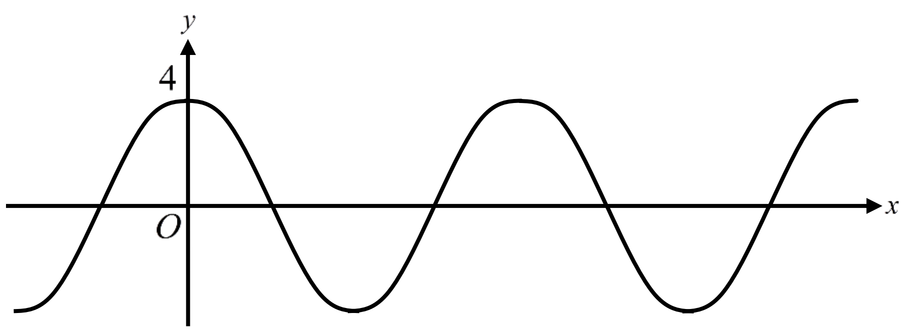
(b) For correct x-intercepts A1

For correct maximum and minimum points A1

For correct domain A1

For sinusoidal curve starting at $(0, 4)$ with correct period A1 N4

[4]



3. (a) (i) The amplitude of f is 4. A1 N1

(ii) The period of f

$= 2\pi \div 1$ (M1) for valid approach

$= 2\pi$ A1 N2

[3]

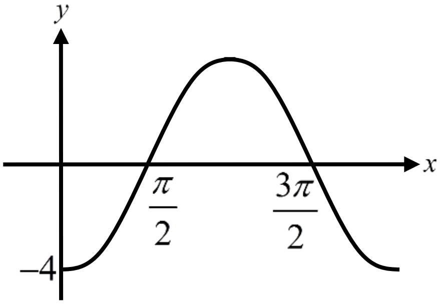
(b) For correct x-intercepts A1

For correct maximum and minimum points A1

For correct domain A1

For sinusoidal curve starting at the $(0, -4)$ with correct period A1 N4

[4]



4. (a) (i) The amplitude of f is 3. A1 N1

(ii) The period of f

$= 2\pi \div \frac{1}{2}$ (M1) for valid approach

$= 4\pi$ A1 N2

[3]

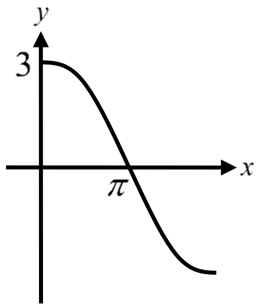
(b) For correct x-intercept A1

For correct maximum and minimum points A1

For correct domain A1

For sinusoidal curve starting at $(0, 3)$ with correct period A1 N4

[4]



Exercise 35

1. $\cos 2x - \cos^2 x + 3\cos x = 3 + \sin^2 x$

$2\cos^2 x - 1 - \cos^2 x + 3\cos x = 3 + 1 - \cos^2 x$	(M2) for valid approach
$2\cos^2 x + 3\cos x - 5 = 0$	A1
$(2\cos x + 5)(\cos x - 1) = 0$	(M1) for factorization
$2\cos x + 5 = 0$ or $\cos x - 1 = 0$	
$\cos x = -\dfrac{5}{2}$ (*Rejected*) or $\cos x = 1$	A1
$x = 0$	A1 N3

[6]

2. $\cos 2x + 7\sin x - 4 = 0$

$1 - 2\sin^2 x + 7\sin x - 4 = 0$	(M1) for valid approach
$2\sin^2 x - 7\sin x + 3 = 0$	A1
$(2\sin x - 1)(\sin x - 3) = 0$	(M1) for factorization
$2\sin x - 1 = 0$ or $\sin x - 3 = 0$	
$\sin x = \dfrac{1}{2}$ or $\sin x = 3$ (*Rejected*)	A1
$x = \dfrac{\pi}{6}$ or $x = \pi - \dfrac{\pi}{6}$	(A1) for correct values
$x = \dfrac{\pi}{6}$ or $x = \dfrac{5\pi}{6}$	A2 N4

[7]

3. $2\sin x = \sin 2x$

$2\sin x = 2\sin x\cos x$	(M1) for valid approach
$2\sin x - 2\sin x\cos x = 0$	(M1) for setting equation
$2\sin x(1 - \cos x) = 0$	A1
$2\sin x = 0$ or $1 - \cos x = 0$	
$\sin x = 0$ or $\cos x = 1$	A1
$x = \pi, 2\pi, 3\pi$ or $x = 2\pi$	
$\therefore x = \pi$, $x = 2\pi$ or $x = 3\pi$	A3 N4

[7]

4. $\cos 2x = \sin 4x$

$\cos 2x = 2\sin 2x \cos 2x$ (M1) for valid approach

$\cos 2x - 2\sin 2x \cos 2x = 0$ (M1) for setting equation

$\cos 2x(1 - 2\sin 2x) = 0$ A1

$\cos 2x = 0$ or $1 - 2\sin 2x = 0$

$\cos 2x = 0$ or $\sin 2x = \frac{1}{2}$ A1

$2x = \frac{\pi}{2}$ or $2x = \frac{\pi}{6}, \frac{5\pi}{6}$

$x = \frac{\pi}{4}$ or $x = \frac{\pi}{12}, \frac{5\pi}{12}$

$\therefore x = \frac{\pi}{12}$, $x = \frac{\pi}{4}$ or $x = \frac{5\pi}{12}$ A3 N4

[7]

Exercise 36

1. (a) $\log_{49}\left(\frac{2+\cos 2x}{6}\right)$

$= \frac{\log_7\left(\frac{2+\cos 2x}{6}\right)}{\log_7 49}$ M1A1

$= \frac{1}{2}\log_7\left(\frac{2+\cos 2x}{6}\right)$ A1

$= \log_7\left(\frac{2+\cos 2x}{6}\right)^{\frac{1}{2}}$

$= \log_7\sqrt{\frac{2+\cos 2x}{6}}$ AG N0

[3]

(b) $\log_7 \cos x = \log_{49}\left(\frac{2+\cos 2x}{6}\right)$

$\log_7 \cos x = \log_7\sqrt{\frac{2+\cos 2x}{6}}$

$\cos x = \sqrt{\frac{2+\cos 2x}{6}}$ M1

$\cos^2 x = \frac{2+\cos 2x}{6}$ A1

$6\cos^2 x = 2 + 2\cos^2 x - 1$ M1

$4\cos^2 x = 1$

$\cos^2 x = \frac{1}{4}$

$\cos x = \frac{1}{2}$ A1

$x = \frac{\pi}{3}$ A1 N3

[5]

2. (a) $\log_4 2\cos 2x$

$= \dfrac{\log_2 2\cos 2x}{\log_2 4}$ M1A1

$= \dfrac{1}{2}\log_2 2\cos 2x$ A1

$= \log_2 (2\cos 2x)^{\frac{1}{2}}$

$= \log_2 \sqrt{2\cos 2x}$ AG N0

[3]

(b) $1+\log_2 \sin x = \log_4 2\cos 2x$

$1+\log_2 \sin x = \log_2 \sqrt{2\cos 2x}$

$1 = \log_2 \sqrt{2\cos 2x} - \log_2 \sin x$

$1 = \log_2 \dfrac{\sqrt{2\cos 2x}}{\sin x}$ M1

$2^1 = \dfrac{\sqrt{2\cos 2x}}{\sin x}$ M1

$2\sin x = \sqrt{2\cos 2x}$

$4\sin^2 x = 2\cos 2x$ A1

$4\sin^2 x = 2(1-2\sin^2 x)$ M1

$4\sin^2 x = 2-4\sin^2 x$

$8\sin^2 x = 2$

$\sin^2 x = \dfrac{1}{4}$

$\sin x = \dfrac{1}{2}$ A1

$x = \dfrac{\pi}{6}$ A1 N3

[6]

3. (a) $\frac{1}{4}+2\log_{81}\cos x$

$=\frac{1}{4}\log_9 9+2\log_{81}\cos x$ M1

$=\frac{1}{4}\log_9 9+2\left(\frac{\log_9\cos x}{\log_9 81}\right)$ M1A1

$=\log_9 9^{\frac{1}{4}}+2\left(\frac{\log_9\cos x}{2}\right)$ A1

$=\log_9 9^{\frac{1}{4}}+\log_9\cos x$

$=\log_9(9^{\frac{1}{4}}\cos x)$

$=\log_9\sqrt{3}\cos x$ AG N0

[4]

(b) $\frac{1}{4}+2\log_{81}\cos x=\log_9\sin 2x$

$\log_9\sqrt{3}\cos x=\log_9\sin 2x$

$\sqrt{3}\cos x=\sin 2x$ M1

$\sqrt{3}\cos x=2\sin x\cos x$ M1

$\sin x=\frac{\sqrt{3}}{2}$ A1

$x=\pi-\frac{\pi}{3}$

$x=\frac{2\pi}{3}$ A1 N2

[4]

4. (a) $\log_{27}\dfrac{\sin 2x}{\sqrt{3}}$

$=\dfrac{\log_3 \dfrac{\sin 2x}{\sqrt{3}}}{\log_3 27}$ M1A1

$=\dfrac{\log_3 \sin 2x-\log_3\sqrt{3}}{3}$ M1

$=\dfrac{\log_3 \sin 2x-\dfrac{1}{2}}{3}$ A1

$=\dfrac{\log_3 \sin 2x}{3}-\dfrac{1}{6}$ AG N0

[4]

(b) $\dfrac{\log_3 \sin 2x}{3}-\dfrac{1}{6}=\log_{27}\sin x$

$\log_{27}\dfrac{\sin 2x}{\sqrt{3}}=\log_{27}\sin x$

$\dfrac{\sin 2x}{\sqrt{3}}=\sin x$ M1

$2\sin x\cos x=\sqrt{3}\sin x$ M1

$\cos x=\dfrac{\sqrt{3}}{2}$ A1

$x=2\pi-\dfrac{\pi}{3}$

$x=\dfrac{5\pi}{3}$ A1 N2

[4]

Exercise 37

1. (a) $h(x)$

$= g(f(x))$ (M1) for composite function

$= 3\cos\left(\frac{f(x)}{4}\right) - 5$ (A1) for substitution

$= 3\cos\left(\frac{2x+3}{4}\right) - 5$

$= 3\cos\left(\frac{x}{2} + \frac{3}{4}\right) - 5$ A1 N3

[3]

(b) The period of h

$= 2\pi \div \frac{1}{2}$ (M1) for valid approach

$= 4\pi$ A1 N2

[2]

(c) $\{y : -8 \le y \le -2\}$ A2 N2

[2]

2. (a) $h(x)$

$= g(f(x))$ (M1) for composite function

$= 4\sin\left(\frac{f(x)}{2}\right) - 3$ (A1) for substitution

$= 4\sin\left(\frac{8x+7}{2}\right) - 3$

$= 4\sin\left(4x + \frac{7}{2}\right) - 3$ A1 N3

[3]

(b) The period of h

$= \frac{2\pi}{4}$ (M1) for valid approach

$= \frac{\pi}{2}$ A1 N2

[2]

(c) $\{y : -7 \le y \le 1\}$ A2 N2

[2]

3. (a) $h(x)$

$= f(g(x))$ (M1) for composite function

$= \frac{3}{2}\left(4\sin\left(\frac{x}{3}\right)+13\right)-1$ (A1) for substitution

$= 6\sin\left(\frac{x}{3}\right)+\frac{37}{2}$ A1 N3

[3]

(b) 6 A2 N2

[2]

(c) $\left\{y : \frac{25}{2} \le y \le \frac{49}{2}\right\}$ A2 N2

[2]

4. (a) $h(x)$

$= f(g(x))$ (M1) for composite function

$= 1-2\left(6\cos\left(\frac{x}{2}\right)+1\right)$ (A1) for substitution

$= -12\cos\left(\frac{x}{2}\right)-1$ A1 N3

[3]

(b) 12 A2 N2

[2]

(c) $\{y : -13 \le y \le 11\}$ A2 N2

[2]

Exercise 38

1. (a) (i) The time required

$= 13.75 - 8.25$ (M1) for valid approach

$= 5.5$ hours A1 N2

(ii) The difference in height

$= 1.8 - 0.4$ (M1) for valid approach

$= 1.4$ m A1 N2

[4]

(b) (i) p

$= \frac{1.8 - 0.4}{2}$ (M1) for valid approach

$= 0.7$ A1 N2

(ii) Period

$= 2(5.5)$ (M1) for valid approach

$= 11$ hours (A1) for correct value

$\therefore q = \frac{2\pi}{11}$ A1 N2

(iii) r

$= \frac{1.8 + 0.4}{2}$ (M1) for valid approach

$= 1.1$ A1 N2

[7]

(c) Recognizing that 9 April 2018 implies $25 \le t < 49$ (M1) for valid approach

t

$= 8.25 + 3(11)$

$= 41.25$ A1

Thus, the time is 16:15. A1 N1

[3]

2. (a) (i) The time required

$= 2(13 - 6.5)$ (M1) for valid approach

$= 13$ hours A1 N2

(ii) The difference in height

$= 4.2 - 1.8$ (M1) for valid approach

$= 2.4$ m A1 N2

[4]

(b) (i) p

$= -\frac{4.2 - 1.8}{2}$ (M1) for valid approach

$= -1.2$ A1 N2

(ii) Period

$= 13$ hours (A1) for correct value

$\therefore q = \frac{2\pi}{13}$ A1 N2

(iii) r

$= \frac{4.2 + 1.8}{2}$ (M1) for valid approach

$= 3$ A1 N2

[6]

(c) Recognizing that 25 August 2018 implies

$36 \le t < 60$ (M1) for valid approach

t

$= 13 + 3(13)$

$= 52$ A1

Thus, the time is 16:00. A1 N1

[3]

3. (a) The time required

$= 2 \times 9$ — (M1) for valid approach

$= 18$ minutes

Hence, the Ferris wheel will first reach a height of 91 m at 9:18. — A1 N2

[2]

(b) (i) p

$= -\frac{91-1}{2}$ — (M1) for valid approach

$= -45$ — A1 N2

(ii) Period

$= 36$ minutes — (A1) for correct value

$\therefore q$

$= \frac{2\pi}{36}$

$= \frac{\pi}{18}$ — A1 N2

(iii) r

$= \frac{1+91}{2}$ — (M1) for valid approach

$= 46$ — A1 N2

[6]

(c) $46 - 45\cos\frac{\pi t}{18} = 60$ — M1

$-14 - 45\cos\frac{\pi t}{18} = 0$ — A1

By considering the graph of the function

$y = -14 - 45\cos\frac{\pi t}{18}$, $t = 82.81262$. — (M1) for valid approach

Thus, the time is 10:22. — A1 N2

[4]

4. (a) (i) p

$= -\frac{73-3}{2}$ (M1) for valid approach

$= -35$ A1 N2

(ii) q

$= \frac{2\pi}{26}$ (M1)(A1) for correct value

$= \frac{\pi}{13}$ A1 N2

(iii) r

$= \frac{73+3}{2}$ (M1) for valid approach

$= 38$ A1 N2

[7]

(b) The height

$= h(56)$

$= -35\cos\left(\frac{\pi}{13}\cdot 56\right)+38$ (M1) for substitution

$= 18.11773386$

$= 18.1\text{ m}$ A1 N2

[2]

(c) $h(t) = 10$

$-35\cos\left(\frac{\pi}{13}t\right)+38=10$ M1

$-35\cos\left(\frac{\pi}{13}t\right)+28=0$ A1

By considering the graph of the function

$y=-35\cos\left(\frac{\pi}{13}t\right)+28$, $t = 75.337174$. (M1) for valid approach

Thus, the time is 14:00. A1 N2

[4]

Chapter 11 Solution

Exercise 39

1. (a) $\frac{1}{2}\times 3\times 9\times \sin A\hat{B}C=\frac{27\sqrt{3}}{4}$ (M1)A1 for substitution

$\sin A\hat{B}C=\frac{\sqrt{3}}{2}$ A1

$A\hat{B}C=\pi-\frac{\pi}{3}$ (M1) for valid approach

$A\hat{B}C=\frac{2\pi}{3}$ A1 N2

[5]

(b) $D\hat{B}C$

$=\pi-A\hat{B}C$

$=\frac{\pi}{3}$ A1

The arc length of the sector BDC

$=(9)\left(\frac{\pi}{3}\right)$ M1

$=3\pi$ cm A1 N2

[3]

2. (a) $\cos B\hat{A}C = \dfrac{10^2 + (10\sqrt{3})^2 - 10^2}{2(10)(10\sqrt{3})}$ (M1)A1 for cosine rule

$\cos B\hat{A}C = \dfrac{\sqrt{3}}{2}$ A1

$B\hat{A}C = \dfrac{\pi}{6}$ A1 N2

[4]

(b) $C\hat{B}D = \dfrac{\pi}{6} + \dfrac{\pi}{6}$ (M1) for valid approach

$C\hat{B}D = \dfrac{\pi}{3}$

The length of arc CD

$= (10)\left(\dfrac{\pi}{3}\right)$

$= \dfrac{10\pi}{3}$ cm (A1) for correct value

The total perimeter

$= \dfrac{10\pi}{3} + 20 + 10\sqrt{3}$ cm A1 N2

[3]

3. (a) $AC^2 = 6^2 + 16^2 - 2 \times 6 \times 16 \times \cos\dfrac{\pi}{3}$ M1A1

$AC^2 = 196$ A1

$AC = \sqrt{196}$

$AC = 14$ cm AG N0

[3]

(b) The area of this shape

$= \dfrac{1}{2}(6)(16)\sin\dfrac{\pi}{3} + \dfrac{1}{2}\pi\left(\dfrac{14}{2}\right)^2$ A2

$= 24\sqrt{3} + \dfrac{49\pi}{2}$ cm^2 A1 N2

[3]

4. (a) $\frac{1}{2}(8)(\text{AC})\sin\frac{\pi}{3}=24\sqrt{3}$ M1A1

$2\sqrt{3}\text{AC}=24\sqrt{3}$ A1

$\text{AC}=\frac{24\sqrt{3}}{2\sqrt{3}}$ M1

$\text{AC}=12\text{ cm}$ AG N0

[4]

(b) $\text{AB}=\sqrt{12^2+8^2-2(12)(8)\cos\frac{\pi}{3}}$ (M1) for cosine rule

$\text{AB}=\sqrt{112}\text{ cm}$

The perimeter of this shape

$=\sqrt{112}+8+\frac{1}{2}(2\pi)(6)$ A1

$=\sqrt{112}+8+6\pi\text{ cm}$ A1 N2

[3]

Exercise 40

1. (a) $\frac{AB}{\sin 48^\circ} = \frac{10}{\sin 114^\circ}$ M1A1

$AB = \frac{10\sin 48^\circ}{\sin 114^\circ}$

$AB = 8.134732862$

$AB = 8.13 \text{ cm}$ A1 N2

[3]

(b) $B\hat{A}C = 180^\circ - 114^\circ - 48^\circ$

$B\hat{A}C = 18^\circ$ (A1) for correct value

$\frac{BC}{\sin 18^\circ} = \frac{10}{\sin 114^\circ}$ M1

$BC = \frac{10\sin 18^\circ}{\sin 114^\circ}$

$BC = 3.382612127$

$BC = 3.38 \text{ cm}$ A1 N2

[3]

2. (a) $B\hat{C}A = \pi - 1.6 - 0.75$ (M1) for valid approach

$\frac{AB}{\sin(\pi - 1.6 - 0.75)} = \frac{21}{\sin 0.75}$ M1

$AB = \frac{21\sin(\pi - 1.6 - 0.75)}{\sin 0.75}$

$AB = 21.91914733$

$AB = 21.9 \text{ cm}$ A1 N2

[3]

(b) The area of ΔABC

$= \frac{1}{2}(AC)(AB)\sin B\hat{A}C$ (M1) for valid approach

$= \frac{1}{2}(21)(21.91914733)\sin 1.6$ A1

$= 230.0529113$

$= 230 \text{ cm}^2$ A1 N2

[3]

3. (a) $\frac{1}{2}(86)(x)\sin 40^\circ = 1900$ (M1)A1 for valid approach

$x = 68.74128537$

$x = 68.7$ cm A1 N2

[3]

(b) $\text{AB} = \sqrt{\begin{array}{l}86^2 + 68.74128537^2 \\ -2(86)(68.74128537)\cos 40^\circ\end{array}}$ (M1)A1 for cosine rule

$\text{AB} = 55.35374433$

$\text{AB} = 55.4$ cm A1 N2

[3]

4. (a) $\frac{1}{2}(35)(54)\sin x^\circ = 892$ (M1)A1 for valid approach

$x = 70.7198401$

$x = 70.7$ A1 N2

[3]

(b) $\text{BC} = \sqrt{35^2 + 54^2 - 2(35)(54)\cos 70.7198401^\circ}$ (M1)A1 for cosine rule

$\text{BC} = 53.78560244$

$\text{BC} = 53.8$ cm A1 N2

[3]

Exercise 41

1.	(a)	The length of major arc ABC	
		$=55(2\pi-2.7)$	(M1)(A1) for substitution
		$=197.0751919$	
		$=197$ cm	A1 N2
			[3]
	(b)	The perimeter of OABC	
		$=197.0751919+55+55$	(M1) for valid approach
		$=307.0751919$	
		$=307$ cm	A1 N2
			[2]
	(c)	The area of OABC	
		$=\frac{1}{2}(55)^2(2\pi-2.7)$	(M1) for valid approach
		$=5419.567777$	
		$=5420\text{ cm}^2$	A1 N2
			[2]
2.	(a)	The length of major arc ABC	
		$=(20)(0.95)$	(M1) for valid approach
		$=19$ cm	A1
		The perimeter of OABC	
		$=19+20+20$	(M1) for valid approach
		$=59$ cm	A1 N3
			[4]
	(b)	The area of OABC	
		$=\frac{1}{2}(20)^2(0.95)$	(M1) for valid approach
		$=190\text{ cm}^2$	A1 N2
			[2]
3.	(a)	$8.6\theta=9.46$	A1
		$\theta=1.1$	A1 N1
			[2]
	(b)	The reflex $A\hat{O}C$	
		$=2\pi-1.1$	(M1)A1 for valid approach
		The area of OADC	
		$=\frac{1}{2}(8.6)^2(2\pi-1.1)$	(M1) for valid approach
		$=191.6741927$	
		$=192\text{ cm}^2$	A1 N3
			[4]

4. (a) $\frac{1}{2}(\text{OC})^2(2) = 14$ A1

$\text{OC}^2 = 14$

$\text{OC} = 3.741657387$

$\text{OC} = 3.74\ \text{cm}$ A1 N1

[2]

(b) The reflex $\text{A}\hat{\text{O}}\text{C}$

$= 2\pi - 2$ (M1)A1 for valid approach

The area of OADC

$= \frac{1}{2}(\sqrt{14})^2(2\pi - 2)$ (M1) for valid approach

$= 29.98229715$

$= 30.0\ \text{cm}^2$ A1 N3

[4]

Exercise 42

1. (a) The required area

$=\frac{1}{2}(125)^2(2.48)$ (A1) for substitution

$=19375\text{ cm}^2$ A1 N2

[2]

(b) The required area

$=\frac{1}{2}(125)^2\sin 2.48$ (A1) for substitution

$=4799.798889$

$=4800\text{ cm}^2$ A1 N2

[2]

(c) The required area

$=19375-4799.798889$ (A1) for correct approach

$=14575.20111$

$=14600\text{ cm}^2$ A1 N2

[2]

2. (a) The required length

$=(1740)(1.4)$ (A1) for substitution

$=2436\text{ cm}$ A1 N2

[2]

(b) $AB=\sqrt{1740^2+1740^2-2(1740)(1740)\cos 1.4}$ (M1)A1 for cosine rule

$AB=2241.877552$

$AB=2240\text{ cm}$ A1 N2

[3]

(c) The required perimeter

$=2436+2241.877552$ (M1) for correct approach

$=4677.877552$

$=4680\text{ cm}$ A1 N2

[2]

3. Let O be the centre of the circle.

$\cos A\hat{O}B = \dfrac{20^2 + 20^2 - 32^2}{2(20)(20)}$ (M1)A1 for cosine rule

$A\hat{O}B = 1.854590436$ A1

The area of the sector AOB

$= \dfrac{1}{2}(20)^2(1.854590436)$

$= 370.9180872$ (A1) for correct value

The area for triangle AOB

$= \dfrac{1}{2}(20)(20)\sin 1.854590436$

$= 192$ (A1) for correct value

The required area

$= 370.9180872 - 192$ (M1) for valid approach

$= 178.9180872$

$= 179 \text{ cm}^2$ A1 N4

[7]

4. Let O be the centre of the circle.

$\cos A\hat{O}B = \dfrac{40^2 + 40^2 - 60^2}{2(40)(40)}$ (M1)A1 for cosine rule

$A\hat{O}B = 1.696124158$ A1

Reflex $A\hat{O}B$

$= 2\pi - 1.696124158$ (M1) for valid approach

$= 4.587061149$ (A1) for correct value

The length of major arc AB

$= (40)(4.587061149)$

$= 183.482446$ A1

The required perimeter

$= 183.482446 + 60$

$= 243.482446$

$= 243 \text{ cm}$ A1 N4

[7]

Exercise 43

1. (a) The bearing of C from E

$=360^\circ-(180^\circ-77^\circ)$ (M1) for valid approach

$=257^\circ$ A1 N2

[2]

(b) $A\hat{C}E$

$=180^\circ-77^\circ$

$=103^\circ$ (A1) for correct value

$A\hat{E}C$

$=180^\circ-103^\circ-51^\circ$ (M1) for valid approach

$=26^\circ$

$\frac{AE}{\sin 103^\circ}=\frac{800}{\sin 26^\circ}$ (M1)(A1) for sine rule

$AE=\frac{800\sin 103^\circ}{\sin 26^\circ}$

$AE=1778.164593$

$AE=1780$ km A1 N2

[5]

(c) $DE=\sqrt{1778.164593^2+1350^2-2(1778.164593)(1350)\cos 51^\circ}$ (M1)A1 for cosine rule

$DE=1401.061804$

$DE=1400$ km A1 N2

[3]

(d) B lies on AC such that $BE\perp AC$. (M1) for valid approach

BE

$=AE\sin B\hat{A}E$

$=1778.164593\sin 51^\circ$

$=1381.893432$ (A1) for correct value

The time required

$=\frac{DE}{62}$

$=\frac{1401.061804}{62}$

$=22.59777103$ h (M1) for valid approach

The speed of the boat

$=\frac{BE}{22.59777103}$ (M1) for valid approach

$=\frac{1381.893432}{22.59777103}$

$=61.15175829$

$=61.2$ km/h A1 N3

[5]

2. (a) $A\hat{D}C$

$= 160^\circ - 90^\circ$

$= 70^\circ$ A1

$\frac{AC}{\sin 70^\circ} = \frac{15}{\sin 58^\circ}$ (M1) for sine rule

$AC = \frac{15 \sin 70^\circ}{\sin 58^\circ}$

$AC = 16.62097866$

$AC = 16.6 \text{ km}$ A1 N2

[3]

(b) $D\hat{A}C$

$= 180^\circ - 70^\circ - 58^\circ$

$= 52^\circ$ (A1) for correct value

The area of the triangle DAC

$= \frac{1}{2}(15)(16.62097866)\sin 52^\circ$ (M1) for valid approach

$= 98.2313244$

$= 98.2 \text{ km}^2$ A1 N2

[3]

(c) The area of the triangle ABC

$= 2(98.2313244)$

$= 196.4626488$ (A1) for correct value

$\frac{1}{2}(16.620979)(BC)\sin 56^\circ = 196.4626488$ (M1)A1 for valid approach

$BC = 28.51538144$

$BC = 28.5 \text{ km}$ A1 N2

[4]

(d) $\frac{DC}{\sin 52^\circ} = \frac{15}{\sin 58^\circ}$ (M1) for sine rule

$DC = \frac{15 \sin 52^\circ}{\sin 58^\circ}$ (A1) for correct value

$BD = \sqrt{DC^2 + BC^2 - 2(DC)(BC)\cos(58^\circ + 56^\circ)}$

$$BD = \sqrt{\left(\frac{15 \sin 52^\circ}{\sin 58^\circ}\right)^2 + 28.51538144^2 - 2\left(\frac{15 \sin 52^\circ}{\sin 58^\circ}\right)(28.51538144)\cos 114^\circ}$$

$BD = 36.47892111$ (A1) for correct value

$\frac{28.51538144}{1} = \frac{36.47892111}{T}$ (M1) for valid approach

$T = 1.279271722$

Therefore, the time taken is 1.28 hours. A1 N3

[5]

3. (a) $A\hat{B}C$

$= 360^\circ - 312^\circ$

$= 48^\circ$ A1

$\frac{AC}{\sin 48^\circ} = \frac{60}{\sin 83^\circ}$ (M1) for sine rule

$AC = 44.9235428$

$AC = 44.9 \text{ km}$ A1 N2

[3]

(b) The area of the triangle ABC

$= \frac{1}{2}(AC)(AB)\sin B\hat{A}C$ (M1) for valid approach

$= \frac{1}{2}(44.9235428)(60)\sin 49^\circ$ A1

$= 1017.126844$

$= 1020 \text{ km}^2$ A1 N2

[3]

(c) The area of the triangle ACD

$= 1.5(1017.126844)$

$= 1525.690266$ (A1) for correct value

$\frac{1}{2}(DC)(AC)\sin\theta^\circ = 1525.690266$ (M1) for valid approach

$\frac{1}{2}(83)(44.9235428)\sin\theta^\circ = 1525.690266$ A1

$\sin\theta^\circ = 0.8183597859$

$\theta^\circ = 54.92093749^\circ$

$\theta^\circ = 54.9^\circ$ A1 N2

[4]

(d) $\frac{BC}{\sin(180^\circ - 48^\circ - 83^\circ)} = \frac{60}{\sin 83^\circ}$

$BC = 45.62263905$ A1

$BD = \sqrt{DC^2 + BC^2 - 2(DC)(BC)\cos(\theta^\circ + 83^\circ)}$ (M1) for cosine rule

$BD = \sqrt{\begin{array}{l} 83^2 + 45.62263905^2 \\ -2(83)(45.62263905) \\ \times\cos(54.92093749^\circ + 83^\circ) \end{array}}$ (A1) for correct formula

$BD = 120.7954013$

$\frac{BD}{\text{Speed of Q}} = \frac{BC + DC}{50}$ (M1) for valid approach

$\frac{120.7954013}{\text{Speed of Q}} = \frac{45.62263905 + 83}{50}$

$\text{Speed of Q} = 46.95728613$

$\text{Speed of Q} = 47.0 \text{ km/h}$ A1 N3

[5]

4. (a)

$B\hat{A}D$
$= 220^\circ - 180^\circ$
$= 40^\circ$ A1
$B\hat{D}A$
$= 180^\circ - 40^\circ - 61^\circ$
$= 79^\circ$ (A1) for correct value
$\frac{BD}{\sin 40^\circ} = \frac{80}{\sin 79^\circ}$ (M1) for sine rule
$BD = 52.38547754$
$BD = 52.4 \text{ km}$ A1 N2

[4]

(b) The area of the triangle ABD

$= \frac{1}{2}(BD)(AB)\sin A\hat{B}D$ (M1) for valid approach
$= \frac{1}{2}(52.38547754)(80)\sin 61^\circ$ A1
$= 1832.694841$
$= 1830 \text{ km}^2$ A1 N2

[3]

(c) The area of the triangle BCD is 1832.694841 km^2.

$\frac{1}{2}(CD)(BD)\sin\theta^\circ = 1832.694841$ (M1)A1 for valid approach
$\frac{1}{2}(72)(52.38547754)\sin\theta^\circ = 1832.694841$ A1
$\sin\theta^\circ = 0.9717996746$
$\theta^\circ = 180^\circ - 76.36074617^\circ$
$\theta^\circ = 103.6392538^\circ$
$\theta^\circ = 104^\circ$ A1 N2

[4]

(d) $BC = \sqrt{CD^2 + BD^2 - 2(CD)(BD)\cos\theta^\circ}$ (M1) for cosine rule

$BC = \sqrt{72^2 + 52.38547754^2 - 2(72)(52.38547754)\cos 103.6392538^\circ}$
$BC = 98.52440125$ (A1) for correct value
The total distance
$= 98.52440125 + 80$
$= 178.52440125$
The minimum time required
$= \frac{178.52440125}{70}$ (M1) for valid approach
$= 2.550348589$ h (A1) for correct value
$= 2$ hours 33 minutes A1 N3

[5]

Exercise 44

1. (a) $\frac{\sin A\hat{C}B}{20.8} = \frac{\sin 1.25}{26.6}$ M1

$\sin A\hat{C}B = 0.742063161$ A1

$A\hat{C}B = 0.8361429666$

$A\hat{C}B = 0.836$ radians A1 N2

[3]

(b) $B\hat{A}C$

$= \pi - 0.8361429666 - 1.25$ (M1) for valid approach

$= 1.055449687$ A1

$\frac{BC}{\sin 1.055449687} = \frac{26.6}{\sin 1.25}$ M1A1

$BC = 24.38948227$

$BC = 24.4$ km A1 N3

[5]

(c) $\cos B\hat{O}C = \frac{14^2 + 14^2 - 24.38948227^2}{2(14)(14)}$ (M1)(A1) for cosine rule

$B\hat{O}C = 2.114683829$ (A1) for correct value

Reflex $B\hat{O}C$

$= 2\pi - 2.114683829$

$= 4.168501478$ (A1) for correct value

The required area

$= \frac{1}{2}(14)^2(4.168501478)$ M1

$= 408.5131448$

$= 409 \text{ cm}^2$ A1 N4

[6]

2. (a) $\dfrac{AC}{\sin 0.873}=\dfrac{43.2}{\sin 1.22}$ M1A1

$AC = 35.24912531$

$AC = 35.2$ cm A1 N2

[3]

(b) $B\hat{A}C$

$= \pi - 0.873 - 1.22$ (M1) for valid approach

$= 1.048592654$ A1

$\dfrac{BC}{\sin 1.048592654}=\dfrac{43.2}{\sin 1.22}$ M1A1

$BC = 39.87053659$

$BC = 39.9$ cm A1 N3

[5]

(c) $\cos B\hat{O}C = \dfrac{23^2+23^2-39.87053659^2}{2(23)(23)}$ (M1) for cosine rule

$B\hat{O}C = 2.097300325$ (A1) for correct value

The area of the sector OBDC

$= \dfrac{1}{2}(23)^2(2.097300325)$ (M1) for valid approach

$= 554.735936$ (A1) for correct value

The area of the triangle OBC

$= \dfrac{1}{2}(23)(23)\sin 2.097300325$

$= 228.6785375$ (A1) for correct value

The required area

$= 554.735936 - 228.6785375$ (M1) for valid approach

$= 326.0573985$

$= 326 \text{ cm}^2$ A1 N4

[7]

3. (a) $\frac{AC}{\sin 0.7} = \frac{11}{\sin 0.37}$ M1A1

$AC = 19.59649377$

$AC = 19.6$ cm A1 N2

[3]

(b) $O\hat{A}C$

$= \pi - 0.37 - 0.7$ (M1) for valid approach

$= 2.071592654$

$\frac{OC}{\sin 2.071592654} = \frac{11}{\sin 0.37}$ M1A1

$OC = 26.68361107$

$OC = 26.7$ cm A1 N3

[4]

(c) The area of the sector OBA

$= \frac{1}{2}(11)^2(0.7)$ (M1) for valid approach

$= 42.35$ (A1) for correct value

The area of the triangle OAC

$= \frac{1}{2}(11)(26.68361107)\sin 0.7$ (M1) for valid approach

$= 94.54529816$ (A1) for correct value

The required area

$= 94.54529816 - 42.35$ (M1) for valid approach

$= 52.19529816$

$= 52.2\text{ cm}^2$ A1 N4

[6]

4. (a) $\frac{\sin A\hat{C}B}{28} = \frac{\sin 1.1}{47}$ M1

$\sin A\hat{C}B = 0.5309320443$ A1

$A\hat{C}B = 0.5597000552$

$A\hat{C}B = 0.560$ A1 N2

[3]

(b) $O\hat{A}C$

$= \pi - 1.1 - 0.5597000552$ (M1) for valid approach

$= 1.481892598$

$\frac{OC}{\sin 1.481892598} = \frac{47}{\sin 1.1}$ M1A1

$OC = 52.52916817$

$OC = 52.5$ cm A1 N3

[4]

(c) The length of the arc ADB

$= (28)(1.1)$ (M1) for valid approach

$= 30.8$ (A1) for correct value

The required perimeter

$= 30.8 + (52.52916817 - 28) + 47$ (M1) for valid approach

$= 102.3291682$

$= 102$ cm A1 N2

[4]

Chapter 12 Solution

Exercise 45

1.	(a)	$\pi r^2 = 9\pi$	(M1) for setting equation	
		$r^2 = 9$		
		$r = 3$ cm	A1 N2	
				[2]
	(b)	12π cm^3	A1 N1	
				[1]
	(c)	The slant height l of the circular cone		
		$= \sqrt{3^2 + 4^2}$	(M1) for valid approach	
		$= 5$		
		The total surface area		
		$= \pi r^2 + \pi rl$	(M1) for valid approach	
		$= \pi(3)^2 + \pi(3)(5)$	(A1) for substitution	
		$= 75.39822369$		
		$= 75.4$ cm^2	A1 N3	
				[4]
2.	(a)	$\pi r^2 = 37$	(M1) for setting equation	
		$r = \sqrt{\dfrac{37}{\pi}}$		
		$r = 3.431831259$		
		$r = 3.43$ cm	A1 N2	
				[2]
	(b)	84.7 cm^3	A2 N2	
				[2]
	(c)	The total surface area		
		$= 2\pi r^2 + \pi r^2$	(M1) for valid approach	
		$= 3\pi r^2$		
		$= 3(37)$	(A1) for substitution	
		$= 111$ cm^2	A1 N3	
				[3]

3. (a) $V=\frac{1}{3}\pi r^2 h$ (M1) for setting equation

$150=\frac{1}{3}\pi r^2(13)$

$r=\sqrt{\frac{450}{13\pi}}$

$r=3.319400418$

$r=3.32$ cm A1 N2

[2]

(b) l

$=\sqrt{3.319400418^2+13^2}$ (M1) for valid approach

$=13.41709429$

$=13.4$ cm A1 N2

[2]

(c) The curved surface area

$=\pi rl$ (M1) for valid approach

$=\pi(3.319400418)(13.41709429)$

$=139.9161959$

$=140\text{ cm}^2$ A1 N2

[2]

4. (a) $A=\pi rl$ (M1) for setting equation

$369\pi=\pi r(41)$

$r=9$ cm A1 N2

[2]

(b) The vertical height h

$=\sqrt{41^2-9^2}$ (M1) for valid approach

$=40$ cm A1 N2

[2]

(c) The volume

$=\frac{1}{3}\pi r^2 h$ (M1) for valid approach

$=\frac{1}{3}\pi(9)^2(40)$

$=1080\pi\text{ cm}^3$ A1 N2

[2]

Exercise 46

1. (a) The volume

$=\frac{1}{3}\pi R^2 H+\pi r^2 h$ (M2) for valid approach

$=\frac{1}{3}\pi(12)^2(16)+\pi(12)^2(5)$ (A1) for substitution

$=4674.689869$

$=4670\ \text{m}^3$ A1 N3

[4]

(b) The slant height of the top

$=\sqrt{12^2+16^2}$ (M1) for valid approach

$=20$

The area

$=\pi rl$ (M1) for valid approach

$=\pi(12)(20)$

$=753.9822369$

$=754\ \text{m}^2$ A1 N2

[3]

2. (a) The volume

$=\frac{1}{3}\pi r^2 h+\frac{2}{3}\pi r^3$ (M2) for valid approach

$=\frac{1}{3}\pi(8)^2(6)+\frac{2}{3}\pi(8)^3$ (A1) for substitution

$=1474.454152$

$=1470\ \text{cm}^3$ A1 N3

[4]

(b) The slant height of the circular cone

$=\sqrt{6^2+8^2}$ (M1) for valid approach

$=10$

The total surface area

$=\pi rl+2\pi r^2$ (M1) for valid approach

$=\pi(8)(10)+2\pi(8)^2$

$=653.4512719$

$=653\ \text{cm}^2$ A1 N2

[3]

3. (a) $V=\frac{2}{3}\pi r^3+\pi r^2 h$ (M2) for setting equation

$54000\pi=\frac{2}{3}\pi r^3+\pi r^2(40)$ (A1) for substitution

$54000=\frac{2}{3}r^3+40r^2$

$\frac{2}{3}r^3+40r^2-54000=0$ (M1) for quadratic equation

$r=30$ m A1 N3

[5]

(b) The area

$=2\pi r^2$ (M1) for valid approach

$=2\pi(30)^2$

$=5654.866776$

$=5650\text{ m}^2$ A1 N2

[2]

4. (a) $A=2\pi r^2+2\pi rh+2\pi r^2$ (M2) for setting equation

$28\pi=4\pi r^2+2\pi r(3)$ (A1) for substitution

$28=4r^2+6r$

$2r^2+3r-14=0$ (M1) for quadratic equation

$(2r+7)(r-2)=0$

$2r+7=0$ or $r-2=0$

$r=-\frac{7}{2}$ (*Rejected*) or $r=2$ mm A1 N3

[5]

(b) The volume

$=\frac{4}{3}\pi r^3+\pi r^2 h$ (M1) for valid approach

$=\frac{4}{3}\pi(2)^3+\pi(2)^2(3)$

$=71.20943348$

$=71.2\text{ mm}^3$ A1 N2

[2]

Exercise 47

1. (a) The volume

$= \frac{2}{3}\pi r^3$ (M1) for valid approach

$= \frac{2}{3}\pi(22)^3$

$= 22301.11905$ (A1) for correct value

$= 22300$

$= 2.23\times10^4 \text{ cm}^3$ A1 N3

[3]

(b) $V = \pi r^2 h$ (M1) for setting equation

$22301.11905 = \pi r^2(26)$ (A1) for substitution

$r^2 = 273.025641$

$r = 16.52348756$

$r = 16.5 \text{ cm}$ A1 N3

[3]

2. (a) The volume

$= \frac{1}{3}Ah$ (M1) for valid approach

$= \frac{1}{3}(8\pi)^2(35)$

$= 7369.304619$ (A1) for correct value

$= 7370$

$= 7.37\times10^3 \text{ cm}^3$ A1 N3

[3]

(b) $V = \frac{4}{3}\pi r^3$ (M1) for setting equation

$7369.304619 = \frac{4}{3}\pi r^3$ (A1) for substitution

$r^3 = 1759.291886$

$r = 12.07200203$

$r = 12.1 \text{ cm}$ A1 N3

[3]

3. (a) The volume

$= \pi r^2 h$ — (M1) for valid approach

$= \pi(7)^2(100)$

$= 4900\pi \text{ cm}^3$ — A1 N2

[2]

(b) $V = 10\left(\frac{2}{3}\pi r^3\right)$ — (M1) for setting equation

$4900\pi = \frac{20}{3}\pi r^3$ — (A1) for substitution

$r^3 = 735$

$r = 9.024623926$ — (A1) for correct value

$r = 9.02$

$r = 9.02 \times 10^1 \text{ cm}$ — A1 N3

[4]

4. (a) The volume

$= \frac{1}{3}\pi r^2 h$ — (M1) for valid approach

$= \frac{1}{3}\pi(27)^2(27)$

$= 20611.9894$ — (A1) for correct value

$= 20600$

$= 2.06 \times 10^4 \text{ cm}^3$ — A1 N3

[3]

(b) $V = 27\left(\frac{4}{3}\pi r^3\right)$ — (M1) for setting equation

$4(20611.9894) = 36\pi r^3$ — (A1) for substitution

$r^3 = 729$

$r = 9$ — A1

The ratio

$= 27 : 9$

$= 3 : 1$ — A1 N3

[4]

Exercise 48

1. (a) The total surface area

$= 4\pi r^2$

$= 4\pi(15)^2$ (A1) for substitution

$= 2827.433388$

$= 2830 \text{ cm}^2$ A1 N2

[2]

(b) $V = 4\pi r^2$

$2827.433388 \times (1+30\%) = 4\pi r^2$ (M2) for setting equation

$r^2 = 292.5$ (M1) for finding r^2

$r = 17.10263138$

$r = 17.1 \text{ cm}$ A1 N2

[4]

2. (a) The total surface area

$= 4\pi r^2$

$= 4\pi(14)^2$ (A1) for substitution

$= 2463.00864$

$= 2460 \text{ cm}^2$ A1 N2

[2]

(b) $V = 4\pi r^2$

$2463.00864 \times (1+15\%) = 4\pi r^2$ (M2) for setting equation

$r^2 = 225.4$ (M1) for finding r^2

$r = 15.01332741$

The percentage increase

$= \dfrac{\frac{4}{3}\pi(15.01332741)^3 - \frac{4}{3}\pi(14)^3}{\frac{4}{3}\pi(14)^3} \times 100\%$ M1

$= 23.32376089\%$

$= 23.3\%$ A1 N3

[5]

3. (a) The total surface area

$= 2\pi r^2 + 2\pi rh$

$= 2\pi(18)^2 + 2\pi(18)(8)$ (A1) for substitution

$= 2940.530724$

$= 2940 \text{ cm}^2$ A1 N2

[2]

(b) Increase in total surface area

$= 2(2rh)$ (M1) for valid approach

$= 2(2)(18)(8)$

$= 576$ (A1) for correct value

The percentage increase

$= \frac{576}{2940.530724} \times 100\%$ M1

$= 19.58830069\%$

$= 19.6\%$ A1 N2

[4]

4. (a) The total surface area

$= \pi r^2 + \pi rl$

$= \pi(7)^2 + \pi(7)(25)$ (A1) for substitution

$= 703.7167544$

$= 704 \text{ cm}^2$ A1 N2

[2]

(b) The vertical height h

$= \sqrt{25^2 - 7^2}$ (M1) for valid approach

$= 24$

Increase in total surface area

$= 2\left(\frac{1}{2}(2r)(h)\right)$ (M1) for valid approach

$= 2rh$

$= 2(7)(24)$

$= 336$ (A1) for correct value

The percentage increase

$= \frac{336}{703.7167544} \times 100\%$ M1

$= 47.74648293\%$

$= 47.7\%$ A1 N3

[5]

Exercise 49

1. (a) The volume

$= \frac{2}{3}\pi r^3$ (M1) for valid approach

$= \frac{2}{3}\pi (10)^3$ (A1) for substitution

$= 2094.395102$

$= 2090 \text{ cm}^3$ A1 N3

[3]

(b) The total surface area

$= 2\pi r^2 + \pi r^2$ (M1) for valid approach

$= 3\pi r^2$

$= 3\pi (10)^2$ (A1) for substitution

$= 942.4777961$

$= 942 \text{ cm}^2$ A1 N3

[3]

(c) $V = \frac{1}{3}\pi r^2 h$ (M1) for setting equation

$4(2094.395102) = \frac{1}{3}\pi \left(\frac{38}{2}\right)^2 (\text{OV})$ (A1) for substitution

$\text{OV} = 22.16066482$

$\text{OV} = 22.2 \text{ cm}$ A1 N3

[3]

(d) The slant height l

$= \sqrt{22.16066482^2 + 19^2}$ (M1) for valid approach

$= 29.19066743$

$\cos \text{A}\hat{\text{V}}\text{B} = \frac{\text{VA}^2 + \text{VB}^2 - \text{AB}^2}{2(\text{VA})(\text{VB})}$ (M1) for cosine rule

$\cos \text{A}\hat{\text{V}}\text{B} = \frac{29.19066743^2 + 29.19066743^2 - 38^2}{2(29.19066743)(29.19066743)}$ (A1) for substitution

$\text{A}\hat{\text{V}}\text{B} = 81.21792258$

$\text{A}\hat{\text{V}}\text{B} = 81.2^\circ$ A1 N2

[4]

(e) The total surface area

$= \pi r^2 + \pi r l$ (M1) for valid approach

$= \pi (19)^2 + \pi (19)(29.19066743)$ (A1) for substitution

$= 2876.513489$

$= 2880 \text{ cm}^2$ A1 N2

[3]

2. (a) The volume

$= A_1h_1$ (M1) for valid approach

$= (260)(100)$

$= 26000 \text{ cm}^3$ A1 N2

[2]

(b) The total surface area

$= 2A_1 + ph_1$ (M1) for valid approach

$= 2(260) + (62)(100)$ (A1) for substitution

$= 6720 \text{ cm}^2$ A1 N3

[3]

(c) $V = \frac{1}{3}A_2h_2$ (M1) for setting equation

$26000 = \frac{1}{3}(\text{AD})^2(40)$ (A1) for substitution

$\text{AD} = 44.15880433$

$\text{AD} = 44.2 \text{ cm}$ A1 N3

[3]

(d) $\tan \text{V}\hat{\text{M}}\text{O} = \frac{\text{OV}}{\text{OM}}$ (M1) for tangent ratio

$\tan \text{V}\hat{\text{M}}\text{O} = \frac{\text{OV}}{\frac{1}{2}\text{AD}}$

$\tan \text{V}\hat{\text{M}}\text{O} = \frac{40}{\frac{1}{2}(44.15880433)}$ (A1) for substitution

$\text{V}\hat{\text{M}}\text{O} = 61.10195875$

$\text{V}\hat{\text{M}}\text{O} = 61.1^\circ$ A1 N3

[3]

(e) OA

$= \sqrt{\text{OM}^2 + \text{AM}^2}$ (M1) for valid approach

$= \sqrt{\text{OM}^2 + \text{OM}^2}$

$= \sqrt{2\left(\frac{1}{2}(44.15880433)\right)^2}$

$= 31.22498999$ (A1) for correct value

$\tan \text{V}\hat{\text{A}}\text{O} = \frac{\text{OV}}{\text{OA}}$ (M1) for tangent ratio

$\tan \text{V}\hat{\text{A}}\text{O} = \frac{40}{31.22498999}$ (A1) for substitution

$\text{V}\hat{\text{A}}\text{O} = 52.02352051$

$\text{V}\hat{\text{A}}\text{O} = 52.0^\circ$ A1 N3

[5]

3. (a) $\sin \text{O}\hat{\text{V}}\text{A} = \dfrac{\text{OA}}{\text{VA}}$ (M1) for sine ratio

$\sin \text{O}\hat{\text{V}}\text{A} = \dfrac{40}{104}$

$\text{O}\hat{\text{V}}\text{A} = 22.61986495$

$\text{O}\hat{\text{V}}\text{A} = 22.6^\circ$ A1 N2

[2]

(b) The total surface area

$= \pi r^2 + \pi r l$ (M1) for valid approach

$= \pi(40)^2 + \pi(40)(104)$ (A1) for substitution

$= 18095.57368$

$= 18100 \text{ cm}^2$ A1 N3

[3]

(c) The vertical height h

$= \sqrt{104^2 - 40^2}$ (M1) for valid approach

$= 96$

The volume

$= \dfrac{1}{3}\pi r^2 h$ (M1) for valid approach

$= \dfrac{1}{3}\pi(40)^2(96)$ (A1) for substitution

$= 160849.5439$

$= 161000 \text{ cm}^3$ A1 N3

[4]

(d) $\tan \text{O}\hat{\text{V}}\text{A} = \dfrac{R}{H}$ (M1) for tangent ratio

$\therefore \dfrac{40}{96} = \dfrac{R}{H}$

$R = \dfrac{5}{12}H$ A1 N2

[2]

(e) $V = \dfrac{1}{3}\pi R^2 H$ (M1) for setting equation

$\dfrac{160849.5439}{2} = \dfrac{1}{3}\pi\left(\dfrac{5}{12}H\right)^2(H)$ (A1) for substitution

$80424.77193 = \dfrac{25}{432}\pi H^3$

$H^3 = 442368$ (M1) for finding H^3

$H = 76.19525049$

$R = \dfrac{5}{12}(76.19525049)$ (M1) for valid approach

$R = 31.74802104$

Thus, $H = 76.2$ cm and $R = 31.7$ cm. A2 N3

[6]

4. (a) $\cos O\hat{V}A = \frac{VO}{VA}$ (M1) for cosine ratio

$\cos O\hat{V}A = \frac{56}{70}$

$O\hat{V}A = 36.86989765$

$O\hat{V}A = 36.9^\circ$ A1 N2

[2]

(b) OA

$= \sqrt{70^2 - 56^2}$ (M1) for valid approach

$= 42$

$AD^2 = OA^2 + OD^2$ (M1) for Pythagoras' Theorem

$AD^2 = OA^2 + OA^2$

$AD^2 = 42^2 + 42^2$ (A1) for substitution

$AD = 59.39696962$

$AD = 59.4\text{ cm}$ A1 N3

[4]

(c) The volume

$= \frac{1}{3}A_1h_1$ (M1) for valid approach

$= \frac{1}{3}(AD)^2(VO)$

$= \frac{1}{3}(59.39696962)^2(56)$ (A1) for substitution

$= 65856\text{ cm}^3$ A1 N3

[3]

(d) $V = \frac{1}{3}A_2h_2$ (M1) for setting equation

$\frac{65856}{2} = \frac{1}{3}(x)(56 \times 2^{-\frac{1}{3}})$ (A1) for substitution

$x = 2222.500732$ (A1) for correct value

y

$= AD^2$

$= 42^2 + 42^2$

$= 3528$ (A1) for correct value

$\therefore x : y$

$= 2222.500732 : 3528$ (M1) for valid approach

$= 1 : 0.630$ A1 N3

[6]

Chapter 13 Solution

Exercise 50

1. $f'(x)$

$=(3)(\cos x)+(3x)(-\sin x)$ (M1) for product rule

$=3(\cos x - x\sin x)$ A2

$f'\left(\frac{3\pi}{2}\right)$

$=3\left(\cos\frac{3\pi}{2}-\frac{3\pi}{2}\sin\frac{3\pi}{2}\right)$ (M1) for substitution

$=3\left(0-\frac{3\pi}{2}(-1)\right)$

$=\frac{9\pi}{2}$

The gradient of the normal

$=\frac{-1}{f'\left(\frac{3\pi}{2}\right)}$ (M1) for negative reciprocal

$=-\frac{2}{9\pi}$ A1 N3

[6]

2. $f'(x)$

$=(-e^{-x})(\cos x)+(e^{-x})(-\sin x)$ (M1) for product rule

$=-e^{-x}(\cos x+\sin x)$ A2

$f'(2\pi)$

$=-e^{-2\pi}(\cos 2\pi+\sin 2\pi)$ (M1) for substitution

$=-e^{-2\pi}$

The gradient of the normal

$=\frac{-1}{f'(2\pi)}$ (M1) for negative reciprocal

$=e^{2\pi}$ A1 N3

[6]

3. $f'(x)$

$= (2x)(-\sin(x^2))$	(M1) for chain rule
$= -2x\sin(x^2)$	A1
$f'(a) = -2a$	(A1) for correct equation
$-2a\sin(a^2) = -2a$	(M1) for valid approach
$\sin(a^2) = 1$	
$a^2 = \frac{\pi}{2}, \frac{5\pi}{2}, \ldots$	(M1) for solving equation
$a = \sqrt{\frac{\pi}{2}}, \sqrt{\frac{5\pi}{2}}$ (*Rejected*), …	
$\therefore a = \sqrt{\frac{\pi}{2}}$	A1 N3

[6]

4. $g'(x)$

$= (2x)(\ln x) + (x^2)\left(\frac{1}{x}\right)$	(M1) for product rule
$= 2x\ln x + x$	
$= x(2\ln x + 1)$	A1
$g'(a) = 3a$	(A1) for correct equation
$a(2\ln a + 1) = 3a$	(M1) for valid approach
$2\ln a + 1 = 3$	
$\ln a = 1$	(M1) for valid approach
$a = e$	A1 N3

[6]

Exercise 51

1. (a) $f'(x) = -5e^{-5x}$ A1 N1

$f''(x) = 25e^{-5x}$ A1 N1

$f^{(3)}(x) = -125e^{-5x}$ A1 N1

[3]

(b) $f^{(n)}(x) = (-5)^n e^{-5x}$ A2 N2

[2]

2. (a) $f'(x) = -\cos x$

$f''(x) = \sin x$

$f^{(3)}(x) = \cos x$

$f^{(4)}(x) = -\sin x$ A2 N2

[2]

(b) $f^{(2n)}(x) = (-1)^{n+1} \sin x$ A3 N3

[3]

3. (a) $g'(x) = nx^{n-1}$

$g''(x) = n(n-1)x^{n-2}$

$g^{(3)}(x) = n(n-1)(n-2)x^{n-3}$

$g^{(4)}(x) = n(n-1)(n-2)(n-3)x^{n-4}$ A2 N2

[2]

(b) $g^{(12)}(x) = \frac{n!}{(n-k)!}x^{n-12}$

$n(n-1)(n-2)(n-3)\cdots(n-11)x^{n-12} = \frac{n!}{(n-k)!}x^{n-12}$ A1

$\frac{n(n-1)(n-2)(n-3)\cdots(n-11)(n-12)!}{(n-12)!} = \frac{n!}{(n-k)!}$ (A1)

$\frac{n!}{(n-12)!} = \frac{n!}{(n-k)!}$

$\therefore k = 12$ A1 N1

[3]

4. (a) $g'(x) = nx^{-1}$

$g''(x) = -nx^{-2}$

$g^{(3)}(x) = 2nx^{-3}$

$g^{(4)}(x) = -6nx^{-4}$ A2 N2

[2]

(b) $g^{(37)}(x) = (k!)nx^{-37}$

$(-1)^{37+1}(1)(2)(3)\cdots(36)nx^{-37} = (k!)nx^{-37}$ A1

$(36!)nx^{-37} = (k!)nx^{-37}$ (A1)

$\therefore k = 36$ A1 N1

[3]

Exercise 52

1. (a) $f'(x)$

$= \dfrac{(x^2-5x+4)(-2)-(-2x)(2x-5)}{(x^2-5x+4)^2}$ M1A2

$= \dfrac{-2x^2+10x-8-(-4x^2+10x)}{(x^2-5x+4)^2}$ (M1)(A1) for expansion

$= \dfrac{2x^2-8}{(x^2-5x+4)^2}$ A1

$= \dfrac{2(x^2-4)}{(x^2-5x+4)^2}$ AG N0

[6]

(b) $f'(x)=0$

$\dfrac{2(x^2-4)}{(x^2-5x+4)^2}=0$ (M1) for setting equation

$x^2-4=0$ A1

$(x+2)(x-2)=0$

$x+2=0$ or $x-2=0$

$x=-2$ or $x=2$ (*Rejected*) (A1) for correct value

$f(-2)$

$= -\dfrac{2(-2)}{(-2)^2-5(-2)+4}$ (M1)A1 for substitution

$= \dfrac{2}{9}$

Thus, the coordinates of B are $\left(-2, \dfrac{2}{9}\right)$. A2 N4

[7]

(c) The line $y=k$ does not meet the graph of f below its minimum point and above its maximum point. (R1) for correct argument

$\therefore \dfrac{2}{9}<k<2$ A2 N3

[3]

2. (a) $f'(x)$

$= \dfrac{(x^2+3x)(6)-(6x+24)(2x+3)}{(x^2+3x)^2}$ M1A2

$= \dfrac{6x^2+18x-(12x^2+66x+72)}{(x^2+3x)^2}$ (M1)(A1) for expansion

$= \dfrac{-6x^2-48x-72}{(x^2+3x)^2}$ A1

$= -\dfrac{6(x^2+8x+12)}{(x^2+3x)^2}$ AG N0

[6]

(b) $f'(x)=0$

$-\dfrac{6(x^2+8x+12)}{(x^2+3x)^2}=0$ (M1) for setting equation

$x^2+8x+12=0$ A1

$(x+6)(x+2)=0$

$x+6=0$ or $x+2=0$

$x=-6$ (*Rejected*) or $x=-2$ (A1) for correct value

$f(-2)$

$= \dfrac{6(-2)+24}{(-2)^2+3(-2)}$ (M1)A1 for substitution

$=-6$

Thus, the coordinates of B are $(-2,-6)$. A2 N4

[7]

(c) The line $y=k$ meets the graph of f below its maximum point and above its minimum point. (R1) for correct argument

$\therefore k \le -6$ and $k \ge -\dfrac{2}{3}$ A2 N3

[3]

3. (a) $f'(x)$

$$=\frac{\left(\cos\left(\frac{\pi x}{2}\right)\right)(0)-(1)\left(-\sin\left(\frac{\pi x}{2}\right)\right)\left(\frac{\pi}{2}\right)}{\left(\cos\left(\frac{\pi x}{2}\right)\right)^2}+0$$ M1A2

$$=\frac{0-\frac{\pi}{2}\left(-\sin\left(\frac{\pi x}{2}\right)\right)}{\cos^2\left(\frac{\pi x}{2}\right)}$$ (M1)(A1) for expansion

$$=\frac{\frac{\pi}{2}\sin\left(\frac{\pi x}{2}\right)}{\cos^2\left(\frac{\pi x}{2}\right)}$$

$$=\frac{\pi\sin\left(\frac{\pi x}{2}\right)}{2\cos^2\left(\frac{\pi x}{2}\right)}$$ AG N0

[5]

(b) $f'(x)=0$

$$\frac{\pi\sin\left(\frac{\pi x}{2}\right)}{2\cos^2\left(\frac{\pi x}{2}\right)}=0$$ (M1) for setting equation

$$\sin\left(\frac{\pi x}{2}\right)=0$$ A1

$\frac{\pi x}{2}=0$ or $\frac{\pi x}{2}=\pi$

$x=0$ (*Rejected*) or $x=2$ (A1) for setting equation

$f(2)$

$$=\frac{1}{\cos\left(\frac{\pi(2)}{2}\right)}+2$$ (M1)A1 for substitution

$$=\frac{1}{-1}+2=1$$

Thus, the coordinates of Q are $(2,1)$. A2 N4

[7]

(c) The line $y=k$ meets the graph of f at two distinct points below its maximum point and above its minimum point. (R1) for correct argument

$\therefore k<1$ and $k>3$ A2 N3

[3]

4. (a) $f'(x)$

$$= \frac{\left(\sin\left(2x-\frac{\pi}{3}\right)\right)(0)-(1)\left(\cos\left(2x-\frac{\pi}{3}\right)\right)(2)}{\left(\sin\left(2x-\frac{\pi}{3}\right)\right)^2} - 0$$ M1A2

$$= \frac{0-2\cos\left(2x-\frac{\pi}{3}\right)}{\sin^2\left(2x-\frac{\pi}{3}\right)}$$ (M1)(A1) for expansion

$$= -\frac{2\cos\left(2x-\frac{\pi}{3}\right)}{\sin^2\left(2x-\frac{\pi}{3}\right)}$$ AG N0

[5]

(b) $f'(x)=0$

$$-\frac{2\cos\left(2x-\frac{\pi}{3}\right)}{\sin^2\left(2x-\frac{\pi}{3}\right)}=0$$ (M1) for setting equation

$$\cos\left(2x-\frac{\pi}{3}\right)=0$$ A1

$$2x-\frac{\pi}{3}=-\frac{\pi}{2} \text{ or } 2x-\frac{\pi}{3}=\frac{\pi}{2}$$

$$2x=-\frac{\pi}{6} \text{ or } 2x=\frac{5\pi}{6}$$

$$x=-\frac{\pi}{12}\ (\textit{Rejected}) \text{ or } x=\frac{5\pi}{12}$$ (A1) for correct value

$$f\left(\frac{5\pi}{12}\right)$$

$$=\frac{1}{\sin\left(2\left(\frac{5\pi}{12}\right)-\frac{\pi}{3}\right)}-3$$ (M1)A1 for substitution

$$=\frac{1}{1}-3$$

$$=-2$$

Thus, the coordinates of Q are $\left(\frac{5\pi}{12},-2\right)$. A2 N4

[7]

(c) 4 A1 N1

[1]

Exercise 53

1. (a) $h(2)$

$= f(g(2))$ (A1) for valid approach

$= f(7)$

$= 10$ A1 N2

[2]

(b) $h'(7)$

$= f'(g(7)) \cdot g'(7)$ (M1) for chain rule

$= f'(7) \cdot g'(7)$

$= (8)(2)$ (A1) for substitution

$= 16$ A1 N2

[3]

2. (a) $h(-4)$

$= \dfrac{f(-4)}{g(-4)}$ (A1) for valid approach

$= \dfrac{-3}{-1}$

$= 3$ A1 N2

[2]

(b) $h'(6)$

$= \dfrac{f'(6)g(6) - f(6)g'(6)}{(g(6))^2}$ (M1) for quotient rule

$= \dfrac{(-2)(6) - (5)(-5)}{6^2}$ (A1) for substitution

$= \dfrac{13}{36}$ A1 N2

[3]

3. $f'(x)$

$= \pi(-\sin \pi x)$

$= -\pi \sin \pi x$ (A1) for chain rule

$g'(x)$

$= \left(\frac{1}{3x-2}\right)(3)$

$= \frac{3}{3x-2}$ (A1) for correct expression

$h'(1)$

$= f'(1)g(1) + f(1)g'(1)$ (M1) for product rule

$= (-\pi \sin \pi(1)) \ln(3(1)-2) + (\cos \pi(1))\left(\frac{3}{3(1)-2}\right)$ A2

$= (0)(0) + (-1)(3)$

$= -3$ A1 N3

[6]

4. $f'(x)$

$= 2x - 0$

$= 2x$ (A1) for valid approach

$g'(x)$

$= (e^{2x})(2)$

$= 2e^{2x}$ (A1) for correct expression

$h'(2)$

$= \frac{f'(2)g(2) - f(2)g'(2)}{(g(2))^2}$ (M1) for quotient rule

$= \frac{(2(2))(e^{2(2)}) - (2^2 - 3)(2e^{2(2)})}{(e^{2(2)})^2}$ A2

$= \frac{(4)(e^4) - (1)(2e^4)}{e^8}$

$= \frac{2e^4}{e^8}$

$= \frac{2}{e^4}$ A1 N3

[6]

Exercise 54

1. $f'(x)$

$= 5(1+2x^2)^4(4x)$

$= 20x(1+2x^2)^4$ A2

The $(r+1)$ th term

$= 20x\left(\binom{4}{r}1^{4-r}(2x^2)^r\right)$ M1

$= 20\binom{4}{r}2^r x^{2r+1}$

$\therefore 2r+1=7$ R1

$2r=6$

$r=3$ (A1) for correct value

The term in x^7

$= 20\binom{4}{3}2^3 x^{2(3)+1}$ (A1) for correct expansion

$= 640x^7$ A1 N3

[7]

2. $f'(x)$

$= 8(4x^3-7)^7(12x^2)$

$= 96x^2(4x^3-7)^7$ A2

The $(r+1)$ th term

$= 96x^2\left(\binom{7}{r}(4x^3)^{7-r}(-7)^r\right)$ M1

$= 96\binom{7}{r}4^{7-r}(-7)^r x^{23-3r}$

$\therefore 23-3r=11$ R1

$-3r=-12$

$r=4$ (A1) for correct value

The term in x^{11}

$= 96\binom{7}{4}4^{7-4}(-7)^4 x^{23-3(4)}$ (A1) for correct expansion

$= 516311040x^{11}$ A1 N3

[7]

3. $g'(x)$

$= 5(x^2+2x)^4(2x+2)$

$= 10(x+1)(x^2+2x)^4$ A2

$= 10(x+1)\begin{pmatrix} \binom{4}{0}(x^2)^4(2x)^0 + \binom{4}{1}(x^2)^3(2x)^1 \\ +\binom{4}{2}(x^2)^2(2x)^2 + \binom{4}{3}(x^2)^1(2x)^3 \\ +\binom{4}{4}(x^2)^0(2x)^4 \end{pmatrix}$ M1

$= 10(x+1)\begin{pmatrix} (1)(x^8)(1) + (4)(x^6)(2x) \\ +(6)(x^4)(4x^2) + (4)(x^2)(8x^3) + (1)(1)(16x^4) \end{pmatrix}$ A1

$= 10(x+1)(x^8+8x^7+24x^6+32x^5+16x^4)$ A1

The term in x^6

$= \left(10((1)(32)+(1)(24))\right)x^6$ M1

$= 560x^6$ A1 N3

[7]

4. $g'(x)$

$= 4\left(2x-\frac{1}{x}\right)^3\left(2-\frac{-1}{x^2}\right)$

$= 4\left(2+\frac{1}{x^2}\right)\left(2x-\frac{1}{x}\right)^3$ A2

$= 4\left(2+\frac{1}{x^2}\right)\begin{pmatrix} \binom{3}{0}(2x)^3\left(-\frac{1}{x}\right)^0 + \binom{3}{1}(2x)^2\left(-\frac{1}{x}\right)^1 \\ +\binom{3}{2}(2x)^1\left(-\frac{1}{x}\right)^2 + \binom{3}{3}(2x)^0\left(-\frac{1}{x}\right)^3 \end{pmatrix}$ M1

$= 4\left(2+\frac{1}{x^2}\right)\begin{pmatrix} (1)(8x^3)(1) + (3)(4x^2)\left(-\frac{1}{x}\right) \\ +(3)(2x)\left(\frac{1}{x^2}\right) + (1)(1)\left(-\frac{1}{x^3}\right) \end{pmatrix}$ A1

$= 4\left(2+\frac{1}{x^2}\right)\left(8x^3-12x+\frac{6}{x}-\frac{1}{x^3}\right)$ A1

The term in x^{-3}

$= \left(4((2)(-1)+(1)(6))\right)x^{-3}$ M1

$= 16x^{-3}$ A1 N3

[7]

Exercise 55

1. (a) $f'(x) = -3x - 2 - e^{-x-2}$ A1 N1

[1]

(b) $f'(x) = 0$ (M1) for valid approach

$x = -4.421713$ and $x = -0.763463$

$x = -4.42$ and $x = -0.763$ A2 N3

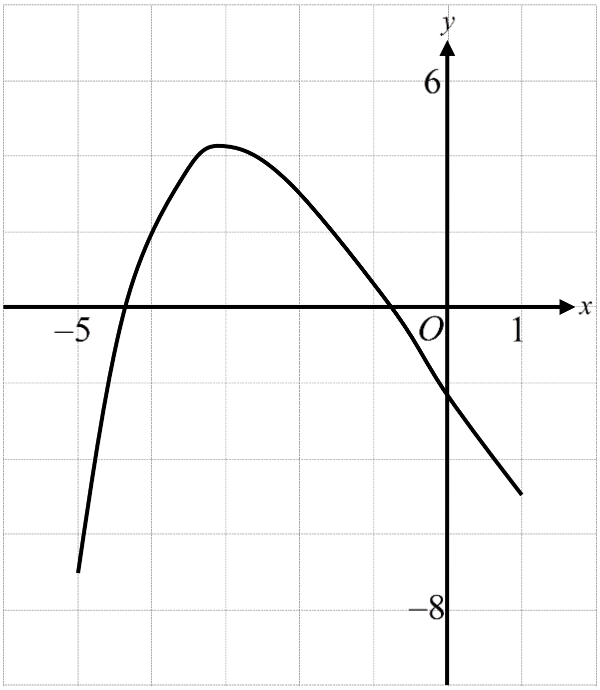
[3]

(c) For correct shape A1

For approximately passing through $(0, -2)$ A1

For approximate range -7.1 to 4.3 A1 N3

[3]



2. (a) $f'(x) = \cos x - \sin x$ A1 N1

[1]

(b) $f'(x) = 0$ (M1) for valid approach

$x = 0.7853982$ and $x = 3.9269908$

$x = 0.785$ and $x = 3.93$ A2 N3

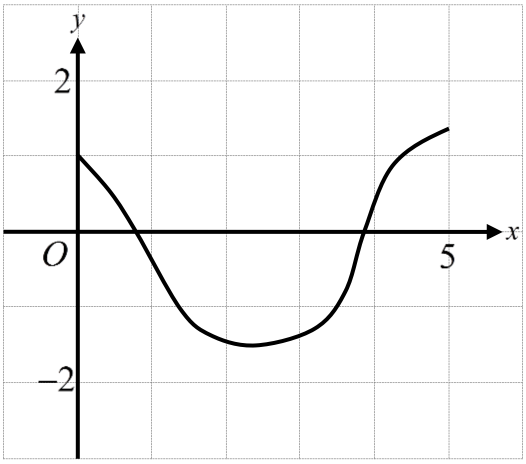
[3]

(c) For correct shape A1

For approximately passing through $(0, 1)$ A1

For approximate range -1.4 to 1.2 A1 N3

[3]



3. (a) $f'(x) = e^x \cos(e^x)$ A1 N1

[1]

(b) $(1.23, -3.29)$ A2 N2

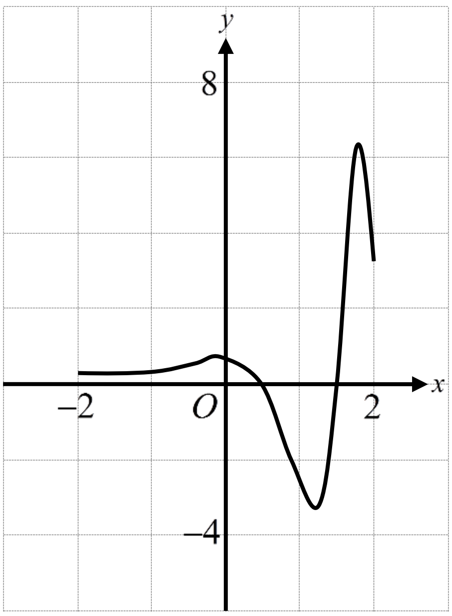
[2]

(c) For correct shape A1

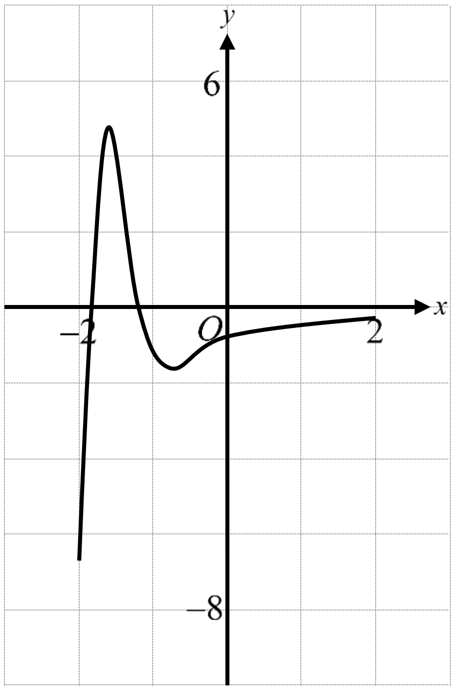
For approximately passing through $(0, 0.5)$ A1

For approximate range -3.3 to 6.4 A1 N3

[3]



4.	(a)	$f'(x) = -e^{-x}\sin(e^{-x})$	A1	N1	
					[1]
	(b)	$(-1.59, 4.81)$	A2	N2	
					[2]
	(c)	For correct shape	A1		
		For approximately passing through $(0, -0.8)$	A1		
		For approximate range -4 to 4.8	A1	N3	
					[3]



Exercise 56

1.	(a)	$f(-2)=29$	(M1) for valid approach
		$a(-2)^3+b(-2)^2+8b(-2)+a=29$	A1
		$-8a+4b-16b+a=29$	
		$-7a-12b=29$	
		$7a+12b=-29$	AG N0
			[2]
	(b)	$f'(x)$	
		$=3ax^2+2bx+8b$	A2
		$f'(-2)=0$	(M1) for valid approach
		$3a(-2)^2+2b(-2)+8b=0$	(A1) for substitution
		$12a+4b=0$	
		$3a+b=0$	A1 N3
			[5]
	(c)	$\begin{cases}7a+12b=-29\\3a+b=0\end{cases}$	(M1) for solving the system
		$a=1,\ b=-3$	A2 N3
			[3]
	(d)	$f'(x)=0$	
		$3(1)x^2+2(-3)x+8(-3)=0$	(M1) for setting equation
		$3x^2-6x-24=0$	
		$x^2-2x-8=0$	
		$(x+2)(x-4)=0$	
		$x=-2$ (*Rejected*) or $x=4$	A1
		$f(4)$	
		$=(1)(4)^3+(-3)(4)^2+8(-3)(4)+1$	
		$=-79$	
		Thus, the coordinates are $(4,-79)$.	A2 N3
			[4]

2.	(a)	$f(-5)=350$	(M1) for valid approach
		$a(-5)^3+b(-5)+b=350$	A1
		$-125a-5b+b=350$	
		$-125a-4b=350$	
		$125a+4b=-350$	AG N0
			[2]
	(b)	$f'(x)$	
		$=3ax^2+b$	A2
		$f'(-5)=0$	(M1) for valid approach
		$3a(-5)^2+b=0$	(A1) for substitution
		$75a+b=0$	A1 N3
			[5]
	(c)	$\begin{cases}125a+4b=-350\\75a+b=0\end{cases}$	(M1) for solving the system
		$a=2$, $b=-150$	A2 N3
			[3]
	(d)	$f'(x)=0$	
		$3(2)x^2-150=0$	(M1) for setting equation
		$6x^2-150=0$	
		$x^2-25=0$	
		$(x+5)(x-5)=0$	
		$x=-5$ (*Rejected*) or $x=5$	A1
		$f(5)$	
		$=2(5)^3+(-150)(5)-150$	
		$=-650$	
		Thus, the coordinates are $(5,-650)$.	A2 N3
			[4]

3.	(a)	$g(\ln 5) = -225 + 126\ln 5$	(M1) for valid approach
		$ae^{2(\ln 5)} + be^{\ln 5} + 126\ln 5 = -225 + 126\ln 5$	A1
		$ae^{\ln 25} + be^{\ln 5} = -225$	(A1) for correct equation
		$25a + 5b = -225$	
		$5a + b = -45$	AG N0
			[3]
	(b)	$g'(x) = 2ae^{2x} + be^{x} + 126$	A2
		$g''(x) = 4ae^{2x} + be^{x}$	A1
		$g''(\ln 5) = 0$	(M1) for valid approach
		$4ae^{2(\ln 5)} + be^{\ln 5} = 0$	(A1) for substitution
		$4ae^{\ln 25} + be^{\ln 5} = 0$	
		$4a(25) + b(5) = 0$	
		$20a + b = 0$	A1 N3
			[6]
	(c)	Valid method for solving system of two equations	
		$\begin{cases} 5a + b = -45 \\ 20a + b = 0 \end{cases}$	(M1) for solving the system
		$a = 3,\ b = -60$	A2 N3
			[3]
	(d)	$g'(x) = 0$	
		$2(3)e^{2x} - 60e^{x} + 126 = 0$	(M1) for setting equation
		$6e^{2x} - 60e^{x} + 126 = 0$	
		$e^{2x} - 10e^{x} + 21 = 0$	
		$(e^{x} - 3)(e^{x} - 7) = 0$	
		$e^{x} = 3$ or $e^{x} = 7$	
		$x = \ln 3$ or $x = \ln 7$	A1
		The horizontal distance	
		$= \ln 7 - \ln 3$	(M1) for valid approach
		$= \ln \frac{7}{3}$	A1 N2
			[4]

4. (a) $g\left(\ln\frac{15}{2}\right)=-\frac{675}{4}+100\ln\frac{15}{2}$ (M1) for valid approach

$ae^{2\left(\ln\frac{15}{2}\right)}+be^{\ln\frac{15}{2}}+100\ln\frac{15}{2}=-\frac{675}{4}+100\ln\frac{15}{2}$ A1

$ae^{\ln\frac{225}{4}}+be^{\ln\frac{15}{2}}=-\frac{675}{4}$ (A1) for correct equation

$\frac{225}{4}a+\frac{15}{2}b+\frac{675}{4}=0$

$225a+30b+675=0$ AG N0

[3]

(b) $g'(x)=2ae^{2x}+be^{x}+100$ A2

$g''(x)=4ae^{2x}+be^{x}$ A1

$g''\left(\ln\frac{15}{2}\right)=0$ (M1) for valid approach

$4ae^{2\left(\ln\frac{15}{2}\right)}+be^{\left(\ln\frac{15}{2}\right)}=0$ (A1) for substitution

$4ae^{\ln\frac{225}{4}}+be^{\ln\frac{15}{2}}=0$

$4a\left(\frac{225}{4}\right)+b\left(\frac{15}{2}\right)=0$

$30a+b=0$ A1 N3

[6]

(c) Valid method for solving system of two equations

$\begin{cases}225a+30b+675=0\\30a+b=0\end{cases}$ (M1) for solving the system

$a=1,\ b=-30$ A2 N3

[3]

(d) $g'(x)=0$

$2(1)e^{2x}-30e^{x}+100=0$ (M1) for setting equation

$e^{2x}-15e^{x}+50=0$

$(e^{x}-5)(e^{x}-10)=0$

$e^{x}=5$ or $e^{x}=10$

$x=\ln 5$ or $x=\ln 10$ A1

$g''(\ln 5)$

$=4e^{2\ln 5}-30e^{\ln 5}$ (M1) for valid approach

$=-50$

<0

$g''(\ln 10)$

$=4e^{2\ln 10}-30e^{\ln 10}$

$=100$

>0

$\therefore g$ attains its local maximum at $x=\ln 5$. (A1) for correct result

$g(\ln 5)$

$= e^{2(\ln 5)} - 30e^{(\ln 5)} + 100(\ln 5)$

$= -125 + 100\ln 5$

Thus, the coordinates are $(\ln 5, -125 + 100\ln 5)$. A2 N3

[6]

Chapter 14 Solution

Exercise 57

1.

$f'(x)$

$= (\cos 3x)(3)$ A1

$= 3\cos 3x$

The slope of L

$= f'(\pi)$ (M1) for valid approach

$= 3\cos 3\pi$

$= -3$ (A1) for correct value

The equation of L:

$y = -3x + b$ (M1) for valid approach

$0 = -3(\pi) + b$ (A1) for substitution

$b = 3\pi$

$\therefore y = -3x + 3\pi$ A1 N3

[6]

2.

$f'(x)$

$= (e^{\pi x})(\pi)$ A1

$= \pi e^{\pi x}$

The slope of L

$= \frac{-1}{f'(1)}$ (M1) for valid approach

$= \frac{-1}{\pi e^{\pi(1)}}$

$= -\frac{1}{\pi e^{\pi}}$ (A1) for correct value

The equation of L:

$y = -\frac{1}{\pi e^{\pi}}x + b$ (M1) for valid approach

$\pi = -\frac{1}{\pi e^{\pi}}(1) + b$ (A1) for substitution

$b = \pi + \frac{1}{\pi e^{\pi}}$

$b = \frac{\pi^2 e^{\pi} + 1}{\pi e^{\pi}}$

$\therefore y = -\frac{1}{\pi e^{\pi}}x + \frac{\pi^2 e^{\pi} + 1}{\pi e^{\pi}}$ A1 N3

[6]

3. $f'(x)$

$= (e^{3x})(3)$ A1

$= 3e^{3x}$

The slope of L

$= f'(k)$ (M1) for valid approach

$= 3e^{3k}$ (A1) for correct value

$\therefore y = 3e^{3k}x - 2e^{3k}$ (M1) for valid approach

$e^{3k} = 3e^{3k}k - 2e^{3k}$ (A1) for substitution

$3e^{3k} = 3e^{3k}k$

$k = 1$ A1 N3

[6]

4. $f'(x)$

$= \frac{1}{2}\left(\frac{1}{x}\right)$ A1

$= \frac{1}{2x}$

The slope of L

$= \frac{-1}{f'(e^{2k})}$ (M1) for valid approach

$= \frac{-1}{\frac{1}{2e^{2k}}}$

$= -2e^{2k}$ (A1) for correct value

$\therefore y = -2e^{2k}x + (2 + 2e^{4k})$ (M1) for valid approach

$k = -2e^{2k}(e^{2k}) + (2 + 2e^{4k})$ (A1) for substitution

$k = -2e^{4k} + 2 + 2e^{4k}$

$k = 2$ A1 N3

[6]

Exercise 58

1. (a) $f(-5)$

$=(-5)^3+6(-5)^2-15(-5)$ — (M1) for substitution

$=-125+150+75$

$=100$ — A1 N2

$f(0)$

$=0^3+6(0)^2-15(0)$

$=0$ — A1 N1

[3]

(b) $f'(x)$

$=3x^2+6(2x)-15(1)$ — A1

$=3x^2+12x-15$

$f'(x)<0$ — R1

$3x^2+12x-15<0$ — (M1) for setting inequality

$x^2+4x-5<0$

$(x+5)(x-1)<0$

$-5<x<1$ — A1 N1

[4]

(c) $f''(x)$

$=3(2x)+12(1)-0$ — A1

$=6x+12$

$f''(x)=0$

$6x+12=0$ — (M1) for setting equation

$6x=-12$

$x=-2$ — A1

$f(-2)$

$=(-2)^3+6(-2)^2-15(-2)$ — (M1) for substitution

$=-8+24+30$

$=46$ — A1

x	$x<-2$	$x=-2$	$x>-2$
$f''(x)$	–	0	+

$f''(x)$ changes its sign at $x=-2$. — (M1) for valid approach

Thus, the coordinates of the point of inflexion on the graph of f are $(-2, 46)$ for $x\ge-5$. — AG N0

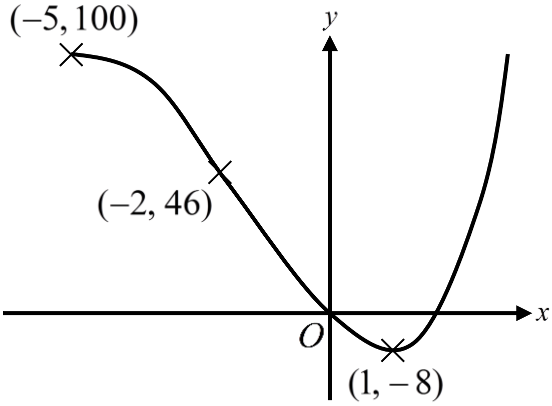
[6]

(d) For concave downward on the left of $(-2, 46)$ and concave upward on the right of $(-2, 46)$ — A1

For passing through $(-5, 100)$ and $(0, 0)$ — A1

For decreasing behavior in $-5<x<1$ — A1 N3

[3]



2. (a) $f(0)$

$= 1 + 9(0) + 3(0)^2 - (0)^3$ (M1) for substitution

$= 1 + 0 + 0 - 0$

$= 1$ A1 N2

$f(3)$

$= 1 + 9(3) + 3(3)^2 - (3)^3$

$= 1 + 27 + 27 - 27$

$= 28$ A1 N1

[3]

(b) $f'(x)$

$= 0 + 9(1) + 3(2x) - 3x^2$ A1

$= 9 + 6x - 3x^2$

$f'(x) > 0$ R1

$9 + 6x - 3x^2 > 0$ (M1) for setting inequality

$3x^2 - 6x - 9 < 0$

$x^2 - 2x - 3 < 0$

$(x+1)(x-3) < 0$

$-1 < x < 3$ A1 N1

[4]

(c) $f''(x)$

$= 0 + 6(1) - 3(2x)$ A1

$= 6 - 6x$

$f''(x) = 0$

$6 - 6x = 0$ (M1) for setting equation

$6 = 6x$

$x = 1$ A1

$f(1)$

$= 1 + 9(1) + 3(1)^2 - (1)^3$ (M1) for substitution

$= 1 + 9 + 3 - 1$

$= 12$ A1

x	$x < 1$	$x = 1$	$x > 1$
$f''(x)$	+	0	–

$f''(x)$ changes its sign at $x = 1$. (M1) for valid approach

Thus, the coordinates of the point of inflexion on the graph of f are $(1, 12)$ for $x \le 3$. AG N0

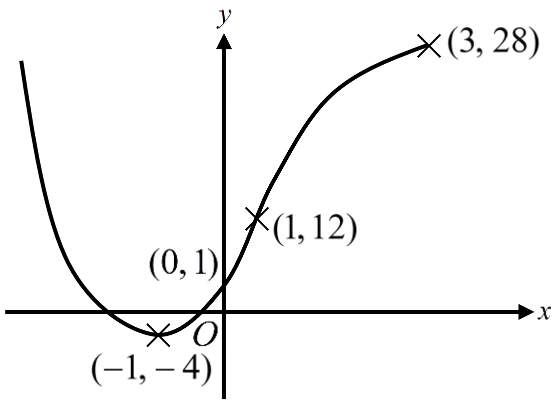
[6]

(d) For concave upward on the left of $(1, 12)$ and concave downward on the right of $(1, 12)$ A1

For passing through $(0, 1)$ and $(3, 28)$ A1

For increasing behavior in $-1 < x < 3$ A1 N3

[3]



3. (a) $f(0)$

$= e^{-k(0)^2}$ (M1) for substitution

$= e^0$

$= 1$ A1 N2

[2]

(b) $f'(x)$

$= (e^{-kx^2})(-2kx)$ A1

$= -2kxe^{-kx^2}$ A1 N1

$f''(x)$

$= (-2k)(e^{-kx^2}) + (-2kx)(e^{-kx^2})(-2kx)$ A2

$= -2ke^{-kx^2}(1-2kx^2)$ A1 N1

[5]

(c) $f''(-1) = 0$

$-2ke^{-k(-1)^2}(1-2k(-1)^2) = 0$ (M1) for setting equation

$-2ke^{-k}(1-2k) = 0$

$1-2k = 0$

$1 = 2k$

$k = \frac{1}{2}$ A1 N2

[2]

(d) $f(-1)$

$= e^{-\frac{1}{2}(-1)^2}$ M1

$= e^{-\frac{1}{2}}$

$= \frac{1}{\sqrt{e}}$

Thus, the coordinates of the point of inflexion on the graph of f are $\left(-1, \frac{1}{\sqrt{e}}\right)$ for $x \le 0$. AG N0

[1]

(e) $f'(x) \ge 0$ R1

$-2\left(\frac{1}{2}\right)xe^{-\frac{1}{2}x^2} \ge 0$ (M1) for setting inequality

$-xe^{-\frac{1}{2}x^2} \ge 0$

$-x \ge 0$ R1

$x \le 0$ AG N0

[3]

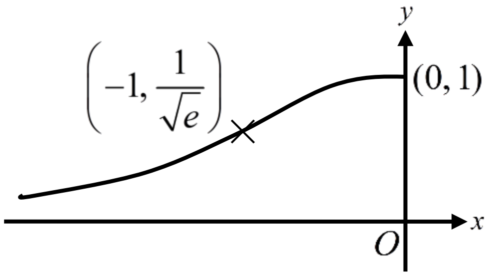
(f) For concave upward on the left of $\left(-1, \frac{1}{\sqrt{e}}\right)$ and

concave downward on the right of $\left(-1, \frac{1}{\sqrt{e}}\right)$ A1

For passing through $(0, 1)$ A1

For increasing behavior in $x < 0$ A1 N3

[3]



4. (a) $f(x) = 0$

$$-\frac{kx}{(x+1)^2} = 0$$ (M1) for setting equation

$kx = 0$

$x = 0$ A1 N2

[2]

(b) $f'(x)$

$$= -\frac{(x+1)^2(k) - (kx)(2)(x+1)}{(x+1)^4}$$ A2

$$= -\frac{k(x+1) - 2kx}{(x+1)^3}$$

$$= -\frac{-kx + k}{(x+1)^3}$$

$$= \frac{kx - k}{(x+1)^3}$$

$$= \frac{k(x-1)}{(x+1)^3}$$ A1 N1

$f''(x)$

$$= \frac{k\left[(x+1)^3(1) - (x-1)(3)(x+1)^2\right]}{(x+1)^6}$$ A2

$$= \frac{k\left[(x+1) - (x-1)(3)\right]}{(x+1)^4}$$

$$= \frac{k(-2x+4)}{(x+1)^4}$$

$$= \frac{-2k(x-2)}{(x+1)^4}$$ A1 N1

[6]

(c) $f(1) = -1$

$-\dfrac{k}{(1+1)^2} = -1$ — (M1) for setting equation

$-k = -4$

$k = 4$ — A1 N2

[2]

(d) $f''(x) = 0$

$\dfrac{-2(4)(x-2)}{(x+1)^4} = 0$ — (M1) for setting equation

$x - 2 = 0$

$x = 2$ — A1

$f(2)$

$= -\dfrac{4(2)}{(2+1)^2}$ — (M1) for substitution

$= -\dfrac{8}{9}$ — A1

x	$x<2$	$x=2$	$x>2$
$f''(x)$	+	0	–

$f''(x)$ changes its sign at $x = 2$. — (M1) for valid approach

Thus, the coordinates of the point of inflexion on the graph of f are $\left(2, -\dfrac{8}{9}\right)$ for $x \ge -1$. — AG N0

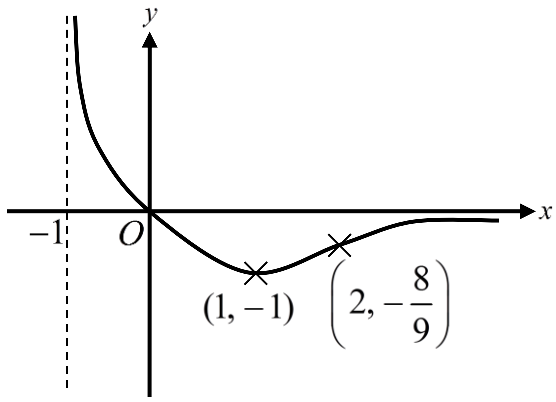
[5]

(e) For concave upward on the left of $\left(2, -\dfrac{8}{9}\right)$ and concave downward on the right of $\left(2, -\dfrac{8}{9}\right)$ — A1

For passing through $(0, 0)$ — A1

For increasing behavior in $-1 < x < 1$ and increasing behavior in $x > 1$ — A1 N3

[3]



Exercise 59

1. (a) P

$= \frac{1}{2}(\text{OB})(\text{OC})\sin\theta + \frac{1}{2}(\text{OA})(\text{OC})\sin(\pi - \theta)$ M1A2

$= \frac{1}{2}(4)(4)\sin\theta + \frac{1}{2}(4)(4)\sin\theta$ M1

$= 16\sin\theta$ AG N0

[4]

(b) Attempt to find $P'(\theta)$

$P'(\theta) = 16\cos\theta$ (M1) for valid approach

$P'(\theta) = 0$ (M1) for setting equation

$16\cos\theta = 0$

$\cos\theta = 0$

$\theta = \frac{\pi}{2}$ A1 N3

By the first derivative test, M1A1

θ	$\theta < \frac{\pi}{2}$	$\theta = \frac{\pi}{2}$	$\theta > \frac{\pi}{2}$
$P'(\theta)$	$+$	0	$-$

Thus, P attains its maximum at $\theta = \frac{\pi}{2}$. R1 N0

[6]

(c) The maximum value of P

$= 16\sin\frac{\pi}{2}$ (M1) for substitution

$= 16$ A1 N2

[2]

(d) $\theta = 0$ and $\theta = \pi$ A2 N2

[2]

2. (a) P

$= \pi(10)^2 - (2(10\cos\theta))(2(10\sin\theta))$ M1A2

$= 100\pi - 400\sin\theta\cos\theta$

$= 100\pi - 200(2\sin\theta\cos\theta)$ M1

$= 100\pi - 200\sin 2\theta$

$= 100(\pi - 2\sin 2\theta)$ AG N0

[4]

(b) Attempt to find $P'(\theta)$

$P'(\theta)$

$= 100(0 - 2(\cos 2\theta)(2))$ (M1) for valid approach

$= -400\cos 2\theta$

$P'(\theta) = 0$ (M1) for setting equation

$-400\cos 2\theta = 0$

$\cos 2\theta = 0$

$2\theta = \frac{\pi}{2}$

$\theta = \frac{\pi}{4}$ A1 N3

By the first derivative test, M1A1

θ	$\theta < \frac{\pi}{4}$	$\theta = \frac{\pi}{4}$	$\theta > \frac{\pi}{4}$
$P'(\theta)$	–	0	+

Thus, P attains its minimum at $\theta = \frac{\pi}{4}$. R1 N0

[6]

(c) The minimum value of P

$= 100\left(\pi - 2\sin 2\left(\frac{\pi}{4}\right)\right)$ (M1) for substitution

$= 100(\pi - 2)$ A1 N2

[2]

(d) $\theta = 0$ and $\theta = \frac{\pi}{2}$ A2 N2

[2]

3. (a) $Q(t)=0$ (M1) for setting equation

$t^3-12t^2+36t=0$

$t(t^2-12t+36)=0$

$t(t-6)^2=0$ (M1) for factorization

$t=0$ and $t=6$ A1 N3

[3]

(b) Attempt to find $Q'(t)$

$Q'(t)$

$=3t^2-12(2t)+36(1)$ (M1) for valid approach

$=3t^2-24t+36$

$Q'(t)=0$ (M1) for setting equation

$3t^2-24t+36=0$

$3(t^2-8t+12)=0$

$3(t-2)(t-6)=0$

$t=2$ or $t=6$ A1 N3

By the first derivative test, M1A1

t	$t<2$	$t=2$	$2<t<6$	$t=6$	$t>6$
$Q'(t)$	+	0	–	0	+

Thus, Q attains its local maximum at $t=2$. R1 N0

[6]

(c) $Q(0)=0$ and $Q(6)=0$ (M1) for valid approach

Thus, the minimum value of Q is 0. A1 N2

[2]

(d) –20 A2 N2

[2]

4. (a) $P(12)$

$=-12^3+9(12)^2-24(12)+720$ M1

$=12(-144+108-24+60)$ M1

$=12(0)$

$=0$

Thus, the t-intercept of P is 12. AG N0

[2]

(b) Attempt to find $P'(t)$

$P'(t)$

$=-3t^2+9(2t)-24(1)+0$ (M1) for valid approach

$=-3t^2+18t-24$

$P'(t)=0$ (M1) for setting equation

$-3t^2+18t-24=0$

$-3(t^2-6t+8)=0$

$-3(t-2)(t-4)=0$

$t=2$ or $t=4$ A1 N3

By the first derivative test, M1A1

t	$t<2$	$t=2$	$2<t<4$	$t=4$	$t>4$
$Q'(t)$	$-$	0	$+$	0	$-$

Thus, Q attains its local minimum at $t=2$. R1 N0

[6]

(c) $P(0)$

$=-0^3+9(0)^2-24(0)+720$ M1

$=720$

$P(4)=704$ and $P(12)=0$ (M1) for valid approach

Thus, the maximum value of P is 720. A1 N2

[3]

(d) 720 A2 N2

[2]

Exercise 60

1. (a) 39.8 m — A2 N2

[2]

(b) The particle first changes direction at 3.7435483 s. — (M1)(A1) for correct value

$s'(t)$

$=(-3t^2)(\cos t)+(-t^3)(-\sin t)+6\cos t$ — (A1) for differentiation

$=t^3\sin t+(6-3t^2)\cos t$

The acceleration at 3.7435483 s

$=s''(3.7435483)$ — (M1) for valid approach

$=-68.94404$

$=-68.9$ cms^{-2} — A1 N2

[5]

2. (a) 1.52 m — A2 N2

[2]

(b) The particle first goes back to O at 1.3932491 s. — (M1)(A1) for correct value

$s'(t)$

$=\cos t-[(4)(\cos t)+(4t)(-\sin t)]$ — (A1) for differentiation

$=4t\sin t-3\cos t$

The acceleration at 1.3932491 s

$=s''(1.3932491)$ — (M1) for valid approach

$=7.8742361$

$=7.87$ cms^{-2} — A1 N2

[5]

3. (a) $v(t)$

$=s'(t)$ — (A1) for differentiation

$=1+(\cos(e^t))(e^t)$

$=1+e^t\cos(e^t)$ — A1 N2

[2]

(b) The particle changes direction for the 4th time at 2.3891023 s. — (M1)(A1) for correct value

The acceleration at 2.3891023 s

$=v'(2.3891023)$ — (M1) for valid approach

$=117.38686$

$=117$ cms^{-2} — A1 N2

[5]

4.	(a)	The particle changes direction for the 1st time and the 3rd time at 2.0318977 s and 7.9806638 s respectively.	(M1)(A1) for correct values
		The amount of time	
		$= 7.9806638 - 2.0318977$	(M1) for valid approach
		$= 5.9487661$	
		$= 5.95\text{ s}$	A1 N2
			[4]
	(b)	The particle is at the maximum distance from O at 11.0854 s.	(A1) for correct value
		$s'(t)$	
		$= \left[(2t)(e^{-t}) + (t^2)(-e^{-t})\right] - \left[(1)(\sin t) + (t)(\cos t)\right]$	(A1) for differentiation
		$= 2te^{-t} - t^2e^{-t} - \sin t - t\cos t$	
		The acceleration at 11.0854 s	
		$= s''(11.0854)$	(M1) for valid approach
		$= -11.21888$	
		$= -11.2\text{ cms}^{-2}$	A1 N2
			[4]

Exercise 61

1. (a) $n(0)$

$= 300e^{0.28(0)}$ (A1) for substitution

$= 300(1)$

$= 300$ A1 N2

[2]

(b) $n'(6)$ (M1) for valid approach

$= 450.70671$

$= 451$ A1 N2

[2]

(c) $n'(k) > 1000$ A1

$300(e^{0.28k})(0.28) > 1000$ M1

$e^{0.28k} > \dfrac{250}{21}$

$k > 8.8462089$ (A1) for correct value

Thus, the least value of k is 9. A1 N2

[4]

2. (a) $V(10)$

$= 10\sqrt{100-10^2}$ (A1) for substitution

$= 10(0)$

$= 0$ A1 N2

[2]

(b) $V'(1)$ (M1) for valid approach

$= 9.8493705$

$= 9.85$ A1 N2

[2]

(c) $V'(k) < 30$ A1

$(k)(\sqrt{100-k^2}) + (1)\left(\dfrac{1}{2\sqrt{100-k^2}}\right)(-2k) < 30$ M1

$k\sqrt{100-k^2} - \dfrac{k}{\sqrt{100-k^2}} < 30$

$k\sqrt{100-k^2} - \dfrac{k}{\sqrt{100-k^2}} - 30 < 0$

$k < 3.2024653$ (A1) for correct value

Thus, the greatest value of k is 3. A1 N2

[4]

3. (a) $p(5)$ — (A1) for correct approach

$= 250\sin(2(5)+3.9)+750$ — (A1) for substitution

$= 993.0018753$

$= 993$ — A1 N3

[3]

(b) $p'(t)=0$ — A1

$250\cos(2t+3.9)(2)+0=0$ — M1

$500\cos(2t+3.9)=0$

$t = 0.4061945$

The value of n

$= (0.4061945)(31)$ — (M1) for substitution

$= 12.5920295$

$= 13$ — A1 N2

[4]

4. (a) $w(13)$ — (A1) for correct approach

$= 145\cos(0.5(13)-5.2)+1020$ — (A1) for substitution

$= 1058.78733$

$= 1059$ — A1 N3

[3]

(b) $w'(t)$

$= 145(-\sin(0.5t-5.2))(0.5)+0$ — A1

$= -72.5\sin(0.5t-5.2)$

$w'(t)$ attains its maximum for the first time when

$t = 7.2584079$. — (A1) for correct value

The value of n

$= (7.2584079)(30)$ — (M1) for substitution

$= 217.752237$

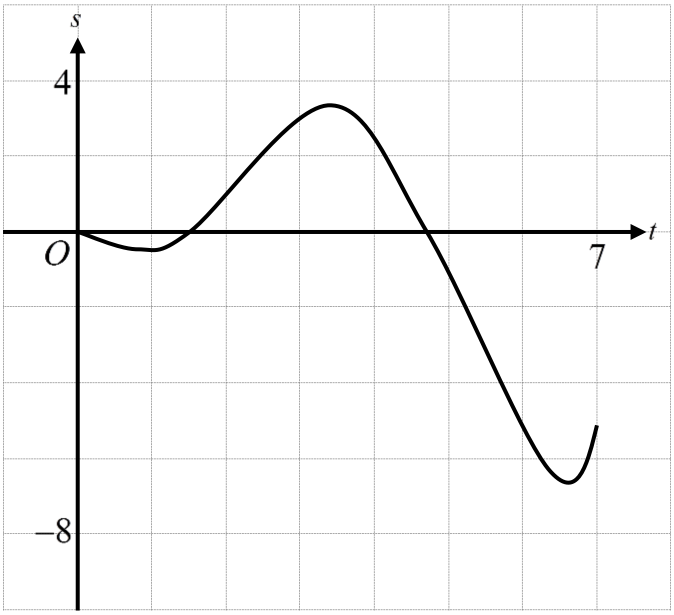
$= 218$ — A1 N2

[4]

Exercise 62

1. (a) For approximately correct shape A1
For approximately correct maximum and minimum points A1
For approximately correct x-intercepts between 1 and 2 and between 4 and 5 A1
For approximately correct endpoints A1 N4

[4]



(b) $v(t)$

$= s'(t)$ (M1) for differentiation

$= (-1)(\cos t) + (-t)(-\sin t)$ (M1) for product rule

$= -\cos t + t\sin t$

The minimum velocity

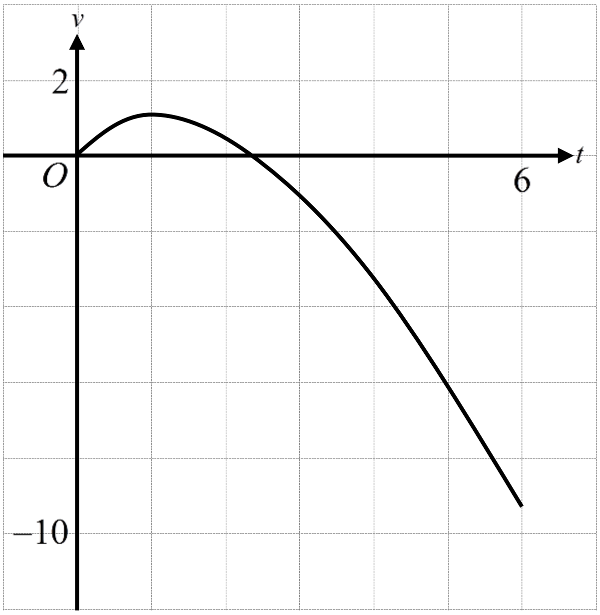
$= -5.100127$

$= -5.10 \text{ ms}^{-1}$ A1 N2

[3]

2. (a) For approximately correct shape A1
For approximately correct maximum point A1
For approximately correct x-intercept between 2 and 3 A1
For approximately correct endpoints A1 N4

[4]



(b) $a(t)$

$= v'(t)$ (M1) for differentiation

$= (2)(\cos\sqrt{t}) + (2t)(-\sin\sqrt{t})\left(\frac{1}{2\sqrt{t}}\right)$ (M1) for product rule

$= 2\cos\sqrt{t} - \sqrt{t}\sin\sqrt{t}$

The maximum acceleration

$= 2\ \text{ms}^{-2}$ A1 N2

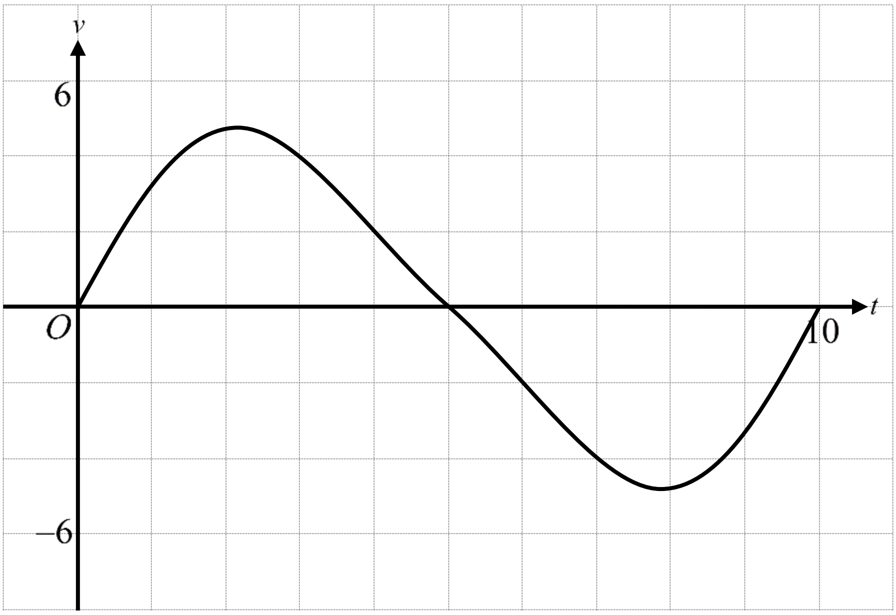
[3]

3. (a) For approximately correct shape A1

For approximately correct maximum and minimum points A1

For correct x-intercepts at 0, 5 and 10 A2 N4

[4]



(b) The acceleration of the particle is zero when its velocity reaches its maximum or minimum. (M1) for valid approach

$t = 2.1132472$ or $t = 7.8867528$

$t = 2.11\,\text{s}$ or $t = 7.89\,\text{s}$ A2 N2

[3]

4. (a) $a(t)$

$= v'(t)$ (M1) for differentiation

$= \frac{\mathrm{d}}{\mathrm{d}t}\left(\frac{1}{3}t^3 - 2t^2\right)$

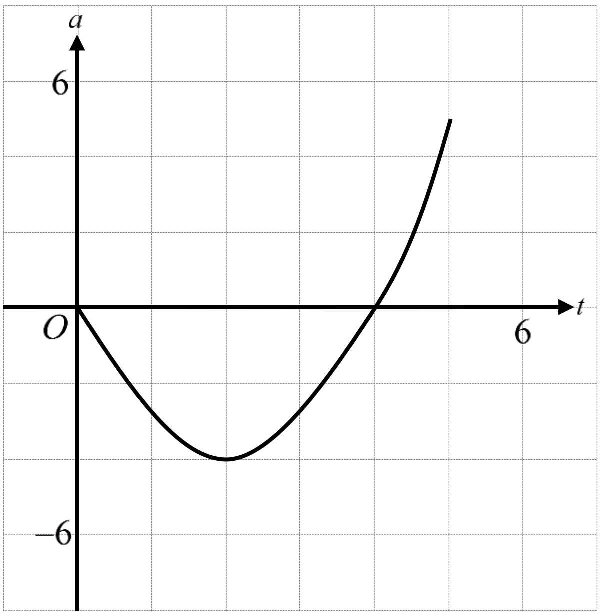
$= \frac{1}{3}(3t^2) - 2(2t)$ (A1) for correct approach

$= t^2 - 4t$ A1 N2

[3]

(b) For approximately correct shape A1
For approximately correct minimum point A1
For correct x-intercept at 0 and 4 A1
For approximately correct endpoint at 5 A1 N4

[4]



Chapter 15 Solution

Exercise 63

1. $\int_3^{10} \frac{5}{5x-1}\,dx$

$= \left[\frac{1}{5} \times 5\ln(5x-1)\right]_3^{10}$ A2

$= \ln(5(10)-1) - \ln(5(3)-1)$ (M1) for substitution

$= \ln 49 - \ln 14$ A1

$= \ln \frac{49}{14}$ (A1) for correct formula

$= \ln \frac{7}{2}$

$\therefore k = \frac{7}{2}$ A1 N3

[6]

2. $\int_0^{6} \frac{4}{4x+1}\,dx$

$= \left[\frac{1}{4} \times 4\ln(4x+1)\right]_0^{6}$ A2

$= \ln(4(6)+1) - \ln(4(0)+1)$ (M1) for substitution

$= \ln 25$ A1

$= \ln 5^2$

$= 2\ln 5$ (A1) for correct formula

$\therefore k = 5$ A1 N3

[6]

3. $\int_0^k \frac{1}{3x+4}\,dx$

$= \left[\frac{1}{3} \times \ln(3x+4)\right]_0^k$ A1

$= \frac{1}{3}\ln(3k+4) - \frac{1}{3}\ln(3(0)+4)$ (M1) for substitution

$= \frac{1}{3}\ln\frac{3k+4}{4}$ A1

$\frac{1}{3}\ln\frac{3k+4}{4} = \ln 2$ (M1) for setting equation

$\ln\frac{3k+4}{4} = 3\ln 2$

$\ln\frac{3k+4}{4} = \ln 2^3$

$\frac{3k+4}{4} = 8$ M1

$3k+4 = 32$

$k = \frac{28}{3}$ A1 N3

[6]

4. $\int_0^k \frac{1}{9-x}\,dx$

$= \left[\frac{1}{-1} \times \ln(9-x)\right]_0^k$ A1

$= -[\ln(9-k) - \ln(9-0)]$ (M1) for substitution

$= \ln\frac{9}{9-k}$ A1

$\ln\frac{9}{9-k} = \ln\frac{k}{2}$ (M1) for setting equation

$\frac{9}{9-k} = \frac{k}{2}$ M1

$18 = 9k - k^2$

$k^2 - 9k + 18 = 0$

$(k-3)(k-6) = 0$

$k = 3$ or $k = 6$ A2 N4

[7]

Exercise 64

1. $f(x) = \int 3x^2(x^3+1)^6 \mathrm{d}x$ (M1) for indefinite integral

Let $u = x^3 + 1$. A1

$\frac{\mathrm{d}u}{\mathrm{d}x} = 3x^2 \Rightarrow \mathrm{d}u = 3x^2\mathrm{d}x$

$\therefore f(x)$

$= \int u^6 \mathrm{d}u$

$= \frac{1}{7}u^7 + C$

$= \frac{1}{7}(x^3+1)^7 + C$ A1

$2 = \frac{1}{7}((-1)^3+1)^7 + C$ M1

$C = 2$ (A1) for correct value

$\therefore f(x) = \frac{1}{7}(x^3+1)^7 + 2$ A1 N4

[6]

2. $f(x) = \int 2x\sin(x^2)\mathrm{d}x$ (M1) for indefinite integral

Let $u = x^2$. A1

$\frac{\mathrm{d}u}{\mathrm{d}x} = 2x \Rightarrow \mathrm{d}u = 2x\mathrm{d}x$

$\therefore f(x)$

$= \int \sin u \mathrm{d}u$

$= -\cos u + C$

$= -\cos(x^2) + C$ A1

$-1 = -\cos(0)^2 + C$ M1

$C = 0$ (A1) for correct value

$\therefore f(x) = -\cos(x^2)$ A1 N4

[6]

3. $f(x)=\int \cos^3 2x \sin 2x \mathrm{d}x$ (M1) for indefinite integral

Let $u=\cos 2x$. A1

$\frac{\mathrm{d}u}{\mathrm{d}x}=-2\sin 2x \Rightarrow -\frac{1}{2}\mathrm{d}u=\sin 2x\mathrm{d}x$ A1

$\therefore f(x)$

$=\int -\frac{1}{2}u^3\mathrm{d}u$

$=-\frac{1}{8}u^4+C$

$=-\frac{1}{8}\cos^4 2x+C$ A1

$3=-\frac{1}{8}\cos^4\left(2\left(\frac{\pi}{2}\right)\right)+C$ M1

$C=\frac{25}{8}$ (A1) for correct value

$\therefore f(x)=-\frac{1}{8}\cos^4 2x+\frac{25}{8}$ A1 N4

[7]

4. $f(x)=\int 4x^3e^{x^4}\mathrm{d}x$ (M1) for indefinite integral

Let $u=x^4$. A1

$\frac{\mathrm{d}u}{\mathrm{d}x}=4x^3 \Rightarrow \mathrm{d}u=4x^3\mathrm{d}x$ A1

$\therefore f(x)$

$=\int e^u\mathrm{d}u$

$=e^u+C$

$=e^{x^4}+C$ A1

$e^{16}-1=e^{2^4}+C$ M1

$C=-1$ (A1) for correct value

$\therefore f(x)=e^{x^4}-1$ A1 N4

[7]

Exercise 65

1. (a) $\int_{9}^{1} 3f(x)\mathrm{d}x$

$= 3\int_{9}^{1} f(x)\mathrm{d}x$ A1

$= -3\int_{1}^{9} f(x)\mathrm{d}x$ A1

$= -3(10)$

$= -30$ AG N0

[2]

(b) $\int_{7}^{9} (x+f(x))\mathrm{d}x + \int_{1}^{7} (x+f(x))\mathrm{d}x$

$= \int_{1}^{9} (x+f(x))\mathrm{d}x$ (A1) for combining integrals

$= \int_{1}^{9} x\mathrm{d}x + \int_{1}^{9} f(x)\mathrm{d}x$ (A1) for separating integrals

$= \left[\frac{1}{2}x^2\right]_1^9 + 10$ A1

$= \frac{1}{2}(9)^2 - \frac{1}{2}(1)^2 + 10$ A1

$= 50$ A1 N3

[5]

2. (a) $\int_{10}^{3} 5f(x)\mathrm{d}x$

$= 5\int_{10}^{3} f(x)\mathrm{d}x$ A1

$= -5\int_{3}^{10} f(x)\mathrm{d}x$ A1

$= -5(-4)$

$= 20$ AG N0

[2]

(b) $\int_{5}^{10} (x+2f(x))\mathrm{d}x + \int_{3}^{5} (x+2f(x))\mathrm{d}x$

$= \int_{3}^{10} (x+2f(x))\mathrm{d}x$ (A1) for combining integrals

$= \int_{3}^{10} x\mathrm{d}x + 2\int_{3}^{10} f(x)\mathrm{d}x$ (A1) for separating integrals

$= \left[\frac{1}{2}x^2\right]_3^{10} + 2(-4)$ A1

$= \frac{1}{2}(10)^2 - \frac{1}{2}(3)^2 - 8$ A1

$= \frac{75}{2}$ A1 N3

[5]

3. (a) $\int_6^0 f(x)\mathrm{d}x$

$= \frac{1}{4}\int_6^0 4f(x)\mathrm{d}x$ A1

$= -\frac{1}{4}\int_0^6 4f(x)\mathrm{d}x$ A1

$= -\frac{1}{4}(12)$

$= -3$ AG N0

[2]

(b) $\int_5^6 (x^2 + f(x))\mathrm{d}x + \int_0^5 (x^2 + f(x))\mathrm{d}x$

$= \int_0^6 (x^2 + f(x))\mathrm{d}x$ (A1) for combining integrals

$= \int_0^6 x^2\mathrm{d}x + \int_0^6 f(x)\mathrm{d}x$ (A1) for separating integrals

$= \left[\frac{1}{3}x^3\right]_0^6 - 3$ A1

$= \frac{1}{3}(6)^3 - \frac{1}{3}(0)^2 - 3$ A1

$= 69$ A1 N3

[5]

4. (a) $\int_2^1 4f(x)\mathrm{d}x$

$= \frac{4}{3}\int_2^1 3f(x)\mathrm{d}x$ A1

$= -\frac{4}{3}\int_1^2 3f(x)\mathrm{d}x$ A1

$= -\frac{4}{3}(6)$

$= -8$ AG N0

[2]

(b) $\int_{1.3}^1 \left(\frac{1}{x}+3f(x)\right)\mathrm{d}x + \int_2^{1.3}\left(\frac{1}{x}+3f(x)\right)\mathrm{d}x$

$= \int_2^1 \left(\frac{1}{x}+3f(x)\right)\mathrm{d}x$ (A1) for combining integrals

$= -\int_1^2 \left(\frac{1}{x}+3f(x)\right)\mathrm{d}x$

$= -\int_1^2 \frac{1}{x}\mathrm{d}x - 3\int_1^2 f(x)\mathrm{d}x$ (A1) for separating integrals

$= -[\ln x]_1^2 - 6$ A1

$= -\ln 2 + \ln 1 - 6$ A1

$= -\ln 2 - 6$ A1 N3

[5]

Exercise 66

1.	(a)	$f'(x)=0$	(M1) for setting equation
		$3x^2-2x-8=0$	
		$(3x+4)(x-2)=0$	M1A1
		$x=2$ or $x=-\frac{4}{3}$ (*Rejected*)	A1
		$\therefore x=2$	A1 N2
			[5]
	(b)	$f(x)$	
		$=\int(3x^2-2x-8)\mathrm{d}x$	(M1) for indefinite integral
		$=x^3-x^2-8x+C$	A3
		$7=0^3-0^2-8(0)+C$	
		$C=7$	(A1) for correct value
		$\therefore f(x)=x^3-x^2-8x+7$	A1 N3
			[6]
	(c)	$g(x)=-f(x+3)-4$	(A1) for transformation
		The local minimum point on the graph of g is the image of A .	(M1) for recognizing image
		The x-coordinate of the required point	
		$=2-3$	M1
		$=-1$	A1 N4

2. (a) $f'(x)=0$ (M1) for setting equation

$36-x^2=0$

$(6+x)(6-x)=0$ M1A1

$x=6$ or $x=-6$ (*Rejected*) A1

$\therefore x=6$ A1 N2

[5]

(b) $f(x)$

$=\int(36-x^2)\mathrm{d}x$ (M1) for indefinite integral

$=36x-\frac{1}{3}x^3+C$ A3

$6=36(0)-\frac{1}{3}(0)^3+C$

$C=6$ (A1) for correct value

$\therefore f(x)=36x-\frac{1}{3}x^3+6$ A1 N3

[6]

(c) $g(x)=f(-(x-6))+5$ (A1) for transformation

The local maximum point on the graph of g is the image of A. (M1) for recognizing image

The x-coordinate of the required point

$=-6+6$ M1

$=0$ A1 N4

[4]

3.	(a)	$f(x)$	
		$=\int (x^2-9)\mathrm{d}x$	(M1) for indefinite integral
		$=\frac{1}{3}x^3-9x+C$	A2
		$0=\frac{1}{3}(0)^3-9(0)+C$	(M1) for substitution
		$C=0$	(A1) for correct value
		$\therefore f(x)=\frac{1}{3}x^3-9x$	A1 N4
			[6]
	(b)	$f'(x)=0$	(M1) for setting equation
		$x^2-9=0$	
		$(x+3)(x-3)=0$	A1
		$x=-3$ (*Rejected*) or $x=3$	M1A1
		When $x=3$, $y=\frac{1}{3}(3)^3-9(3)=-18$	M1
		Thus, the coordinates of A are $(3,-18)$.	A1 N2
			[6]
	(c)	$g(x)=2f(x-1)+4$	(A1) for transformation
		The local minimum point on the graph of g is the image of A.	(M1) for recognizing image
		The coordinates of the required point	
		$=(3+1, 2(-18)+4)$	(M1) for valid approach
		$=(4,-32)$	A1 N4
			[4]

4. (a) $f(x)$

$= \int (-2x-4)\mathrm{d}x$ — (M1) for indefinite integral

$= -x^2 - 4x + C$ — A2

$-5 = -(1)^2 - 4(1) + C$ — (M1) for substitution

$C = 0$ — (A1) for correct value

$\therefore f(x) = -x^2 - 4x$ — A1 N4

[6]

(b) $f'(x) = 0$ — (M1) for setting equation

$-2x - 4 = 0$

$x = -2$ — A1

When $x = -2$, $y = -(-2)^2 - 4(-2) = 4$ — M1

Thus, the coordinates of A are $(-2, 4)$. — A1 N2

[4]

(c) $g(x) = -f(3x)$ — (A1) for transformation

The local minimum point on the graph of g is the image of A. — (M1) for recognizing image

The coordinates of the required point

$= (-2 \div 3, -4)$ — (M1) for valid approach

$= \left(-\frac{2}{3}, -4\right)$ — A1 N4

[4]

Chapter 16 Solution

Exercise 67

1. The area of R

$= \int_0^3 f(x)\mathrm{d}x$ — (A1) for definite integral

$= \int_0^3 \frac{4x}{x^2+1}\mathrm{d}x$

Let $u = x^2+1$ $\frac{\mathrm{d}u}{\mathrm{d}x} = 2x$ $2\mathrm{d}u = 4x\mathrm{d}x$	$x=3 \Rightarrow u = 3^2+1=10$ $x=0 \Rightarrow u = 0^2+1=1$

$= \int_1^{10} \frac{1}{u}\cdot 2\mathrm{d}u$ — (A2) for substitution

$= 2\int_1^{10} \frac{1}{u}\mathrm{d}u$

$= 2[\ln u]_1^{10}$ — A1

$= 2(\ln 10 - \ln 1)$ — (M1) for substitution

$= 2\ln 10$ — A1 N3

[6]

2. The area of R

$= \int_0^{\ln 2} f(x)\mathrm{d}x$ — (A1) for definite integral

$= \int_0^{\ln 2} (e^{2x}+1)\mathrm{d}x$

$= \left[\frac{1}{2}e^{2x}+x\right]_0^{\ln 2}$ — A2

$= \left(\frac{1}{2}e^{2\ln 2}+\ln 2\right) - \left(\frac{1}{2}e^{2(0)}+0\right)$ — (M1) for substitution

$= \left(\frac{1}{2}e^{\ln 4}+\ln 2\right) - \left(\frac{1}{2}(1)+0\right)$

$= \left(\frac{1}{2}(4)+\ln 2\right) - \frac{1}{2}$ — (M1) for substitution

$= \frac{3}{2}+\ln 2$ — A1 N3

[6]

3. The area of $R=\dfrac{\sqrt{3}}{4}$

$\int_0^k g(x)\mathrm{d}x=\dfrac{\sqrt{3}}{4}$ (A1) for correct equation

$\int_0^k \cos 2x\mathrm{d}x=\dfrac{\sqrt{3}}{4}$

$\left[\dfrac{1}{2}\sin 2x\right]_0^k=\dfrac{\sqrt{3}}{4}$ A1

$\dfrac{1}{2}\sin 2k-\dfrac{1}{2}\sin 2(0)=\dfrac{\sqrt{3}}{4}$ (M1) for substitution

$\dfrac{1}{2}\sin 2k=\dfrac{\sqrt{3}}{4}$ A1

$\sin 2k=\dfrac{\sqrt{3}}{2}$

$2k=\dfrac{\pi}{3}$ or $2k=\pi-\dfrac{\pi}{3}$ (M1) for valid approach

$k=\dfrac{\pi}{6}$ or $k=\dfrac{\pi}{3}$ (*Rejected*) A1 N3

[6]

4. The area of $R=\dfrac{1}{2\pi}$

$\int_k^1 g(x)\mathrm{d}x=\dfrac{1}{2\pi}$ (A1) for correct equation

$\int_k^1 \sin \pi x\mathrm{d}x=\dfrac{1}{2\pi}$

$\left[-\dfrac{1}{\pi}\cos \pi x\right]_k^1=\dfrac{1}{2\pi}$ A1

$-\dfrac{1}{\pi}\cos\pi(1)-\left(-\dfrac{1}{\pi}\cos k\pi\right)=\dfrac{1}{2\pi}$ (M1) for substitution

$-\dfrac{1}{\pi}(-1)+\dfrac{1}{\pi}\cos k\pi=\dfrac{1}{2\pi}$ A1

$\dfrac{1}{\pi}\cos k\pi=-\dfrac{1}{2\pi}$

$\cos k\pi=-\dfrac{1}{2}$

$k\pi=\dfrac{2\pi}{3}$ (M1) for valid approach

$k=\dfrac{2}{3}$ A1 N3

[6]

Exercise 68

1. (a) $f(-x)$

$=\frac{1}{4}((-x)^2+a)$ M1A1

$=\frac{1}{4}(x^2+a)$

$=f(x)$ AG N0

[2]

(b) $f'(x)$

$=\frac{1}{4}(2x+0)$ A1

$=\frac{1}{2}x$

$f'(x)=0$ M1

$\frac{1}{2}x=0$

$x=0$

$\therefore 2-a=0$ M1

$a=2$ AG N0

[3]

(c) $f''(x)$

$=\frac{1}{2}(1)$ A1

$=\frac{1}{2}$

$f''(x)$ does not change sign for $-6\le x\le 6$. R1

Thus, there is no point of inflexion. AG N0

[2]

(d) The area of the shaded region

$=\int_2^4 f(x)dx$ (A1) for definite integral

$=\int_2^4 \frac{1}{4}(x^2+2)dx$

$=\frac{1}{4}\left[\frac{1}{3}x^3+2x\right]_2^4$ A1

$=\frac{1}{4}\left(\left(\frac{1}{3}(4)^3+2(4)\right)-\left(\frac{1}{3}(2)^3+2(2)\right)\right)$ (M1) for substitution

$=\frac{1}{4}\left(\frac{64}{3}+8-\frac{8}{3}-4\right)$

$=\frac{17}{3}$ A1 N3

[4]

(e) $\frac{17}{3}$ A2 N2

[2]

2. (a) $f(-x)$

$= \frac{a(-x)^3}{(-x)^4+1}$ M1A1

$= \frac{-ax^3}{x^4+1}$

$= -f(x)$ AG N0

[2]

(b) $f'(x) = 0$

$\frac{ax^2(3-x^4)}{(x^4+1)^2} = 0$ (M1) for setting equation

$ax^2(3-x^4) = 0$

$x^2(\sqrt{3}+x^2)(\sqrt{3}-x^2) = 0$ (A1) for factorization

$x^2(\sqrt{3}+x^2)(3^{\frac{1}{4}}+x)(3^{\frac{1}{4}}-x) = 0$

$x^2 = 0$, $3^{\frac{1}{4}}+x = 0$ or $3^{\frac{1}{4}}-x = 0$

$x = 0$ (*Rejected*), $x = -3^{\frac{1}{4}}$ or $x = 3^{\frac{1}{4}}$ A1

$f(-3^{\frac{1}{4}})$

$= \frac{a(-3^{\frac{1}{4}})^3}{3+1}$

$= -\frac{3^{\frac{3}{4}}}{4}a$ A1

$f(3^{\frac{1}{4}})$

$= -f(-3^{\frac{1}{4}})$

$= \frac{3^{\frac{3}{4}}}{4}a$ A1

Thus, the coordinates of the maximum point and the minimum point are $\left(3^{\frac{1}{4}}, \frac{3^{\frac{3}{4}}}{4}a\right)$ and

$\left(-3^{\frac{1}{4}}, -\frac{3^{\frac{3}{4}}}{4}a\right)$. A1 N4

[6]

(c) The area of the shaded region

$= \int_1^2 f(x)\mathrm{d}x$ (A1) for definite integral

$= \int_1^2 \frac{ax^3}{x^4+1}\mathrm{d}x$

Let $u = x^4 + 1$ $\frac{\mathrm{d}u}{\mathrm{d}x} = 4x^3$ $\frac{1}{4}\mathrm{d}u = x^3\mathrm{d}x$	$x = 2 \Rightarrow u = 2^4 + 1 = 17$ $x = 1 \Rightarrow u = 1^4 + 1 = 2$

$= a\int_2^{17} \frac{1}{u} \cdot \frac{1}{4}\mathrm{d}u$ A2

$= \frac{a}{4}\int_2^{17} \frac{1}{u}\mathrm{d}u$

$= \frac{a}{4}\left[\ln u\right]_2^{17}$ A1

$= \frac{a}{4}(\ln 17 - \ln 2)$ (M1) for substitution

$= \frac{a}{4}\ln\frac{17}{2}$ A1 N3

[6]

(d) $\frac{a}{4}\ln 2$ A2 N2

[2]

3. (a) $g(x)$

$= f(x-3)$

$= (x-3)^3 + 9(x-3)^2 + 15(x-3) + 7$ M1

$= x^3 - 9x^2 + 27x - 27 + 9(x^2 - 6x + 9) + 15x - 45 + 7$ A2

$= x^3 - 9x^2 + 27x - 27 + 9x^2 - 54x + 81 + 15x - 45 + 7$

$= x^3 - 12x + 16$ AG N0

[3]

(b) $g'(x)$

$= 3x^2 - 12(1) + 0$ A1

$= 3x^2 - 12$

$g''(x)$

$= 3(2x) - 0$ A1

$= 6x$

$g''(x) = 0$ (M1) for setting equation

$6x = 0$

$x = 0$

$g(0)$

$= 0^3 - 12(0) + 16$ (M1) for substitution

$= 16$

x	$x<0$	$x=0$	$x>0$
$g''(x)$	–	0	+

$g''(x)$ changes its sign at $x = 0$. (M1) for valid approach

Thus, the coordinates of the point of inflexion of $g(x)$ are $(0, 16)$. A1 N3

[6]

(c) $(-3, 16)$ A2 N2

[2]

(d) The area of the shaded region

$= \int_{-3}^{0} g(x)\mathrm{d}x$ (A1) for definite integral

$= \int_{-3}^{0} (x^3 - 12x + 16)\mathrm{d}x$

$= \left[\frac{1}{4}x^4 - 12\left(\frac{1}{2}x^2\right) + 16x\right]_{-3}^{0}$ A1

$= \left[\frac{1}{4}x^4 - 6x^2 + 16x\right]_{-3}^{0}$

$= \left(\frac{1}{4}(0)^4 - 6(0)^2 + 16(0)\right)$

$-\left(\frac{1}{4}(-3)^4 - 6(-3)^2 + 16(-3)\right)$ (M1) for substitution

$= (0 - 0 + 0) - \left(\frac{81}{4} - 54 - 48\right)$

$= 81.75$ A1 N3

[4]

(e) 81.75 A2 N2

[2]

4. (a) $g(x)$

$= f(x+2)$

$= -2(x+2)^3 + 12(x+2)^2 - 18(x+2)$ M1

$= -2(x^3 + 6x^2 + 12x + 8) + 12(x^2 + 4x + 4) - 18x - 36$ A2

$= -2x^3 - 12x^2 - 24x - 16 + 12x^2 + 48x + 48 - 18x - 36$

$= -2x^3 + 6x - 4$ AG N0

[3]

(b) $g'(x)$

$= -2(3x^2) + 6(1) - 0$ A1

$= -6x^2 + 6$

$g''(x)$

$= -6(2x) + 0$ A1

$= -12x$

$g''(x) = 0$ (M1) for setting equation

$-12x = 0$

$x = 0$

$g(0)$

$= -2(0)^3 + 6(0) - 4$ (M1) for substitution

$= -4$

x	$x<0$	$x=0$	$x>0$
$g''(x)$	+	0	–

$g''(x)$ changes its sign at $x = 0$. (M1) for valid approach

Thus, the coordinates of the point of inflexion of $g(x)$ are $(0, -4)$. A1 N3

[6]

(c) The area of the shaded region

$= -\int_{-2}^{0} g(x)\mathrm{d}x$ (A1) for definite integral

$= \int_{-2}^{0} (2x^3 - 6x + 4)\mathrm{d}x$

$= \left[2\left(\frac{1}{4}x^4\right) - 6\left(\frac{1}{2}x^2\right) + 4x\right]_{-2}^{0}$ A1

$= \left[\frac{1}{2}x^4 - 3x^2 + 4x\right]_{-2}^{0}$

$= \left(\frac{1}{2}(0)^4 - 3(0)^2 + 4(0)\right) - \left(\frac{1}{2}(-2)^4 - 3(-2)^2 + 4(-2)\right)$ (M1) for substitution

$= (0 - 0 + 0) - (8 - 12 - 8)$

$= 12$ A1 N3

[4]

(d)	(i)	-2	A2	N2
	(ii)	24	A2	N2

[4]

Exercise 69

1. (a) (i) $f'(a)\times -4=-1$ (M1) for valid approach

$f'(a)=\frac{1}{4}$ A1 N2

(ii) $f'(x)=\frac{1}{2}x^{-\frac{1}{2}}$ (M1) for valid approach

$f'(a)=\frac{1}{2\sqrt{a}}$

$\therefore \frac{1}{2\sqrt{a}}=\frac{1}{4}$ A1

$\sqrt{a}=2$

$a=4$ AG N0

(iii) $\frac{\sqrt{4}-0}{4-b}=-4$ (M1) for setting equation

$\frac{2}{4-b}=-4$

$-\frac{1}{2}=4-b$

$b=\frac{9}{2}$ A1 N2

(iv) The equation of [PQ]:

$y=-4x+c$

$0=-4\left(\frac{9}{2}\right)+c$ (M1) for substitution

$c=18$

$\therefore y=-4x+18$ A1 N2

[8]

(b) The required area

$=\int_0^4 f(x)\mathrm{d}x+\frac{(18-4)(2-0)}{2}$ M1A1

$=\int_0^4 x^{\frac{1}{2}}\mathrm{d}x+\frac{(14)(2)}{2}$

$=\left[\frac{2}{3}x^{\frac{3}{2}}\right]_0^4+14$ A2

$=\frac{2}{3}(4)^{\frac{3}{2}}-\frac{2}{3}(0)^{\frac{3}{2}}+14$ (M1) for substitution

$=\frac{16}{3}-0+14$

$= \frac{58}{3}$ A1 N3

[6]

2. (a) (i) $f'(x) = 2x$ (M1) for valid approach

$\therefore f'(a) = 2a$ A1 N2

$f(a) = a^2$

The gradient of [PQ]

$= f'(a)$

$= \frac{f(a) - 0}{a - 1}$ (M1) for substitution

$= \frac{a^2}{a - 1}$ A1 N2

(ii) $2a = \frac{a^2}{a - 1}$ (M1) for setting equation

$2a^2 - 2a = a^2$

$a^2 = 2a$ A1

$a = 2$ AG N0

(iii) The equation of [PQ]:

$y = 4x + c$

$0 = 4(1) + c$ (M1) for substitution

$c = -4$

$\therefore y = 4x - 4$ A1 N2

[8]

(b) The required area

$= \int_0^2 f(x)\mathrm{d}x - \frac{(2-1)(4-0)}{2}$ M1A1

$= \int_0^2 x^2\mathrm{d}x - \frac{(1)(4)}{2}$

$= \left[\frac{1}{3}x^3\right]_0^2 - 2$ A2

$= \frac{1}{3}(2)^3 - \frac{1}{3}(0)^3 - 2$ (M1) for substitution

$= \frac{8}{3} - 0 - 2$

$= \frac{2}{3}$ A1 N3

[6]

3. (a) (i) $f'(x) = \frac{1}{2}x^{-\frac{1}{2}}$ M1

$f'(h) = \frac{1}{2\sqrt{h}}$

$f(h) = h^{\frac{1}{2}}$ A1

The equation of [AB]:

$y = \frac{1}{2\sqrt{h}}x + c$ M1

$\sqrt{h} = \frac{1}{2\sqrt{h}}(h) + c$ M1

$\sqrt{h} = \frac{1}{2}\sqrt{h} + c$

$c = \frac{1}{2}\sqrt{h}$

$\therefore y = \frac{1}{2\sqrt{h}}x + \frac{1}{2}\sqrt{h}$ AG N0

(ii) $\frac{\sqrt{h}-0}{h-b} = \frac{1}{2\sqrt{h}}$ (M1) for setting equation

$\frac{\sqrt{h}}{h-b} = \frac{1}{2\sqrt{h}}$

$2h = h - b$ (M1) for valid approach

$b = -h$ A1 N3

[7]

(b) The required area

$= \frac{(h-(-h))(\sqrt{h}-0)}{2} - \int_0^h f(x)\mathrm{d}x$ M1A1

$= \frac{(2h)(\sqrt{h})}{2} - \int_0^h x^{\frac{1}{2}}\mathrm{d}x$

$= h\sqrt{h} - \left[\frac{2}{3}x^{\frac{3}{2}}\right]_0^h$ A2

$= h\sqrt{h} - \left(\frac{2}{3}(h)^{\frac{3}{2}} - \frac{2}{3}(0)^{\frac{3}{2}}\right)$ M1

$= h\sqrt{h} - \left(\frac{2}{3}h\sqrt{h} - 0\right)$ A1

$= \frac{1}{3}h\sqrt{h}$ AG N0

[6]

4. (a) (i) $f'(x) = 3x^2$ M1

$f'(h) = 3h^2$

$f(h) = h^3$ A1

The equation of [AB]:

$y = \frac{-1}{3h^2}x + c$ M1A1

$h^3 = \frac{-1}{3h^2}(h) + c$ M1

$h^3 = -\frac{1}{3h} + c$

$c = h^3 + \frac{1}{3h}$ A1

$c = \frac{3h^4 + 1}{3h}$

$\therefore y = -\frac{1}{3h^2}x + \frac{3h^4 + 1}{3h}$ AG N0

(ii) $\frac{h^3 - 0}{h - b} = -\frac{1}{3h^2}$ (M1) for setting equation

$\frac{h^3}{h - b} = -\frac{1}{3h^2}$

$-3h^5 = h - b$ (M1) for valid approach

$b = 3h^5 + h$ A1 N3

[9]

(b) The required area

$= \frac{(h - (3h^5 + h))(0 - h^3)}{2} + \left(-\int_h^0 f(x)\mathrm{d}x\right)$ M1A1

$= \frac{(-3h^5)(-h^3)}{2} - \int_h^0 x^3 \mathrm{d}x$

$= \frac{3}{2}h^8 - \left[\frac{1}{4}x^4\right]_h^0$ A2

$= \frac{3}{2}h^8 - \left(\frac{1}{4}(0)^4 - \frac{1}{4}h^4\right)$ M1

$= \frac{3}{2}h^8 - \left(0 - \frac{1}{4}h^4\right)$

$= \frac{3}{2}h^8 + \frac{1}{4}h^4$ A1

$= \frac{1}{4}h^4(6h^4 + 1)$ AG N0

[6]

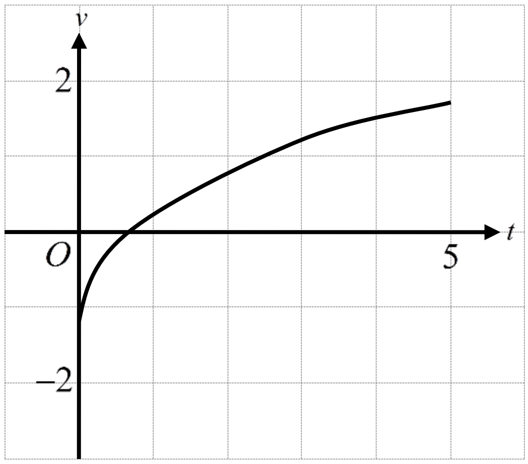
Exercise 70

1. (a) For approximately correct shape A1

For correct x-intercept at 0.7 A1

For approximately correct endpoints A1 N3

[3]



(b) The total distance travelled

$= \int_0^5 |v(t)|\,dt$ (M2) for valid approach

$= \int_0^5 |\ln(t+0.3)|\,dt$ (A1) for correct formula

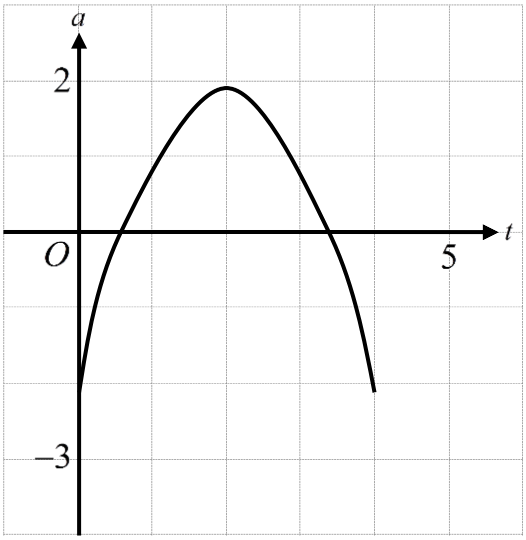
$= 4.877655148$

$= 4.88$ m A1 N3

[4]

2. (a) For approximately correct shape A1
For approximately correct x-intercepts between 0 and 1 and between 3 and 4 A1
For approximately correct endpoints and maximum point A1 N3

[3]



(b) $v(t)$

$= \int a(t)\mathrm{d}t$ (M1) for valid approach

$= \int (-(t-2)^2 + 1.9)\mathrm{d}t$

$= \int (-t^2 + 4t - 2.1)\mathrm{d}t$ (A1) for correct formula

$= -\frac{1}{3}t^3 + 4\left(\frac{1}{2}t^2\right) - 2.1t + C$ A1

$= -\frac{1}{3}t^3 + 2t^2 - 2.1t + C$

$v(0) = 0$ (M1) for substitution

$0 = -\frac{1}{3}(0)^3 + 2(0)^2 - 2.1(0) + C$

$C = 0$

$\therefore v = -\frac{1}{3}t^3 + 2t^2 - 2.1t$ A1 N3

[5]

3. (a) $v(t)$

$= \int a(t)\mathrm{d}t$ — (M1) for valid approach

$= \int \frac{2t}{t^2+1}\mathrm{d}t$

Let $u = t^2 + 1$
$\frac{\mathrm{d}u}{\mathrm{d}t} = 2t$
$\mathrm{d}u = 2t\mathrm{d}t$

$= \int \frac{1}{u}\mathrm{d}u$ — A1

$= \ln u + C$ — A1

$= \ln(t^2+1) + C$

$v(0) = 0$

$0 = \ln(0^2+1) + C$ — (M1) for substitution

$C = 0$

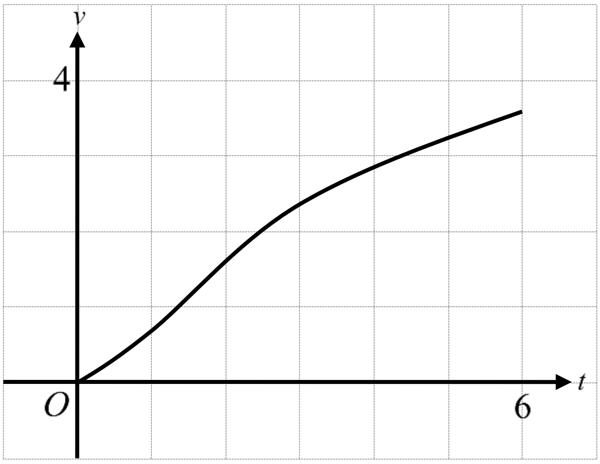
$\therefore v = \ln(t^2+1)$ — A1 N3

[5]

(b) For approximately correct shape — A1

For approximately correct endpoints — A1 N2

[2]



4. (a) $s(t)$

$= \int v(t)\mathrm{d}t$ — (M1) for valid approach

$= \int 2t\cos(t^2)\mathrm{d}t$

Let $u = \sin(t^2)$
$\frac{\mathrm{d}u}{\mathrm{d}t} = 2t\cos(t^2)$
$\mathrm{d}u = 2t\cos(t^2)\mathrm{d}t$

$= \int \mathrm{d}u$ — A1

$= u + C$ — A1

$= \sin(t^2) + C$

$s(0) = 1$

$1 = \sin(0^2) + C$ — (M1) for substitution

$C = 1$

$\therefore s = \sin(t^2) + 1$ — A1 N3

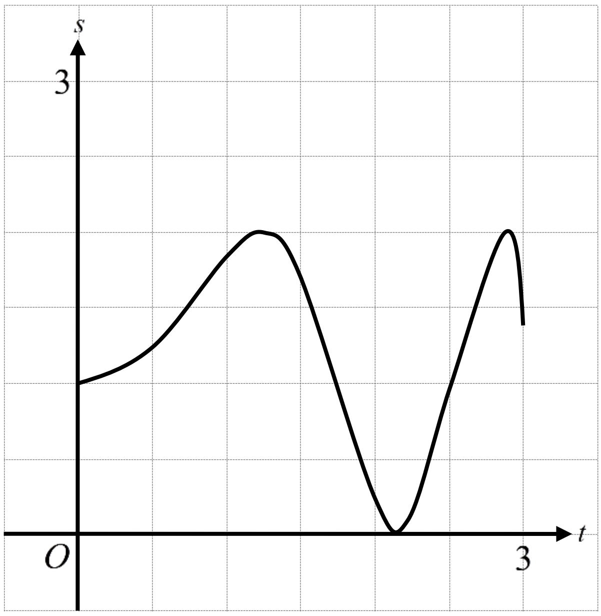
[5]

(b) For approximately correct shape — A1

For approximately correct x-intercept between 2 and 3 — A1

For approximately correct endpoints and maximum points — A1 N3

[3]



Exercise 71

1. (a)	The initial velocity		
	$= v(0)$	(M1) for valid approach	
	$= -(0-4)^3$		
	$= 64 \text{ ms}^{-1}$	A1 N2	
			[2]
(b)	$v(t) = -27$	R1	
	$-(t-4)^3 = -27$		
	$t-4=3$	(A1) for correct approach	
	$t=7$	A1 N2	
			[3]
(c)	The total distance travelled		
	$= \int_0^7 \lvert v(t) \rvert \, dt$	(M1) for valid approach	
	$= \int_0^7 \lvert -(t-4)^3 \rvert \, dt$	(A1) for correct formula	
	$= 84.24999949$		
	$= 84.2 \text{ m}$	A1 N3	
			[3]
(d)	$a(t)$		
	$= v'(t)$	M1	
	$= -3(t-4)^2(1)$	A2	
	$= -3(t-4)^2$	AG N0	
			[3]
(e)	$v(t) < 0$ and $a(t) < 0$		
	$t > 4$ and $t \neq 4$	R2	
	$\therefore t > 4$	A2 N2	
			[4]

2. (a) $s(t)$

$= \int v(t)\mathrm{d}t$ (M1) for valid approach

$= \int (-2t^3 + 12t^2 - 24t + 16)\mathrm{d}t$

$= -2\left(\frac{1}{4}t^4\right) + 12\left(\frac{1}{3}t^3\right) - 24\left(\frac{1}{2}t^2\right) + 16t + C$ A1

$= -\frac{1}{2}t^4 + 4t^3 - 12t^2 + 16t + C$

$0 = -\frac{1}{2}(0)^4 + 4(0)^3 - 12(0)^2 + 16(0) + C$ (M1) for substitution

$C = 0$

$\therefore s = -\frac{1}{2}t^4 + 4t^3 - 12t^2 + 16t$ A1 N2

[4]

(b) The displacement

$= s(3.3)$ (M1) for valid approach

$= -\frac{1}{2}(3.3)^4 + 4(3.3)^3 - 12(3.3)^2 + 16(3.3)$

$= 6.57195 \text{ m}$ A1 N2

[2]

(c) The total distance travelled

$= \int_0^{3.3} |v(t)|\,\mathrm{d}t$ (M1) for valid approach

$= \int_0^{3.3} |-2t^3 + 12t^2 - 24t + 16|\,\mathrm{d}t$ (A1) for correct formula

$= 9.428049981$

$= 9.43 \text{ m}$ A1 N3

[3]

(d) $a(t)$

$= v'(t)$ M1

$= -2(3t^2) + 12(2t) - 24(1) + 0$ A2

$= -6t^2 + 24t - 24$

$= -6(t^2 - 4t + 4)$ M1

$= -6(t-2)^2$ AG N0

[4]

(e) $v(t) > 0$ and $a(t) < 0$

$t < 2$ and $t \neq 2$ R2

$\therefore t < 2$ A2 N2

[4]

3. (a) $s(t)$

$= \int v(t)\mathrm{d}t$ — (M1) for valid approach

$= \int \pi \cos \pi t \mathrm{d}t$

Let $u = \sin \pi t$
$\frac{\mathrm{d}u}{\mathrm{d}t} = \pi \cos \pi t$
$\mathrm{d}u = \pi \cos \pi t \mathrm{d}t$

$= \int \mathrm{d}u$ — A1

$= u + C$ — A1

$= \sin \pi t + C$

$s(0) = 1$

$1 = \sin 0 + C$ — (M1) for substitution

$C = 1$

$\therefore s = \sin \pi t + 1$ — A1 N3

[5]

(b) $s(t) = 0$ — R1

$\sin \pi t + 1 = 0$

$\sin \pi t = -1$ — (A1) for correct formula

$\pi t = \frac{3\pi}{2}$ or $\pi t = \frac{7\pi}{2}$

$t = \frac{3}{2}$ or $t = \frac{7}{2}$ — A2 N3

[4]

(c) $a(t)$

$= v'(t)$ — M1

$= \pi(-\sin \pi t)(\pi)$ — A2

$= -\pi^2 \sin \pi t$

$a(t) > 0$

$-\pi^2 \sin \pi t > 0$ — R1

$\sin \pi t < 0$ — A1

$\pi < \pi t < 2\pi$ or $3\pi < \pi t < 4\pi$

$1 < t < 2$ or $3 < t < 4$ — A1 N4

[6]

(d) 5 — A1 N1

[1]

4. (a) $v(t)$

$= \int a(t)\mathrm{d}t$ M1

$= \int (t-2)(4t^2 - 25t + 38)\mathrm{d}t$

$= \int (4t^3 - 33t^2 + 88t - 76)\mathrm{d}t$

$= 4\left(\frac{1}{4}t^4\right) - 33\left(\frac{1}{3}t^3\right) + 88\left(\frac{1}{2}t^2\right) - 76t + C$ A2

$= t^4 - 11t^3 + 44t^2 - 76t + C$

$48 = 0^4 - 11(0)^3 + 44(0)^2 - 76(0) + C$ M1

$C = 48$

$s(t)$

$= \int v(t)\mathrm{d}t$ M1

$= \int (t^4 - 11t^3 + 44t^2 - 76t + 48)\mathrm{d}t$

$= \frac{1}{5}t^5 - 11\left(\frac{1}{4}t^4\right) + 44\left(\frac{1}{3}t^3\right) - 76\left(\frac{1}{2}t^2\right) + 48t + D$ A2

$= \frac{1}{5}t^5 - \frac{11}{4}t^4 + \frac{44}{3}t^3 - 38t^2 + 48t + D$

$0 = \frac{1}{5}(0)^5 - \frac{11}{4}(0)^4 + \frac{44}{3}(0)^3 - 38(0)^2 + 48(0) + D$ M1

$D = 0$

$\therefore s(t) = \frac{1}{5}t^5 - \frac{11}{4}t^4 + \frac{44}{3}t^3 - 38t^2 + 48t$ AG N0

[8]

(b) $s(t) < 0$ and $a(t) < 0$

$t > 1.0780361$ and

$(t < 2$ or $2.6096118 < t < 3.6403882)$ R2

$\therefore 1.0780361 < t < 2$ or $2.6096118 < t < 3.6403882$

$1.08 < t < 2$ or $2.61 < t < 3.64$ A2 N2

[4]

(c) (i) 3 A1 N1

(ii) 2 A2 N2

[3]

Exercise 72

1. (a) (i) $x = 1.2950976$ and $x = 5.1674957$

$x = 1.30$ and $x = 5.17$ A2 N2

(ii) A

$= \int_{1.2950976}^{5.1674957} (f(x) - g(x))\mathrm{d}x$ (M1) for valid approach

$= \int_{1.2950976}^{5.1674957} (-0.5x^2 + 3x - 2 - 2e^{-0.5x})\mathrm{d}x$ A2

$= 5.366549025$

$= 5.37$ A1 N3

[6]

(b) $f'(x) = g'(x)$ (M1) for setting equation

$-0.5(2x) + 3(1) - 0 = 2e^{-0.5x}(-0.5)$ A2

$-x + 3 = -e^{-0.5x}$

$e^{-0.5x} - x + 3 = 0$ (A1) for correct equation

$x = 3.2017227$

$x = 3.20$ A1 N4

[5]

(c) $\int_0^a g'(x)\mathrm{d}x = 2(e^{-4} - 1)$

$g(a) - g(0) = 2(e^{-4} - 1)$ R1

$2e^{-0.5a} - 2e^{-0.5(0)} = 2e^{-4} - 2$ (M1) for substitution

$2e^{-0.5a} - 2 = 2e^{-4} - 2$

$2e^{-0.5a} = 2e^{-4}$

$-0.5a = -4$ A1

$a = 8$ A1 N3

[4]

2. (a) $f(x)=g(x)$ (M1) for setting equation

$0.1x^2-0.24x+0.174=\sin\left(\frac{\pi}{4}x\right)$

$0.1x^2-0.24x+0.174-\sin\left(\frac{\pi}{4}x\right)=0$

$x=0.173017$ or $x=3.3467439$ (A2) for correct values

A

$=\int_{0.173017}^{3.3467439}(g(x)-f(x))\mathrm{d}x$ (M1) for valid approach

$=\int_{0.173017}^{3.3467439}(\sin\left(\frac{\pi}{4}x\right)-(0.1x^2-0.24x+0.174))\mathrm{d}x$ A1

$=1.909710794$

$=1.91$ A1 N4

[6]

(b) $f'(x)>g'(x)$ (M1) for setting inequality

$0.1(2x)-0.24(1)+0>\left(\cos\left(\frac{\pi}{4}x\right)\right)\left(\frac{\pi}{4}\right)$ A2

$0.2x-0.24-\frac{\pi}{4}\cos\left(\frac{\pi}{4}x\right)>0$ (A1) for correct inequality

$x>1.8035377$

$\therefore 1.80<x\leq 4$ A1 N4

[5]

(c) $\int_a^2 f'(x)\mathrm{d}x=\frac{8}{125}$

$f(2)-f(a)=\frac{8}{125}$ R1

$(0.1(2)^2-0.24(2)+0.174)$

$-(0.1a^2-0.24a+0.174)=\frac{8}{125}$ (M1) for substitution

$-0.08-0.1a^2+0.24a=\frac{8}{125}$

$0.1a^2-0.24a+0.144=0$ A1

$a=1.2$ A1 N3

[4]

3. (a) $f(x)=g(x)$ (M1) for setting equation

$$x^3-11x^2+38x-40=\frac{2}{3}x-2$$

$$x^3-11x^2+\frac{112}{3}x-38=0$$

$x=1.8871981$, $x=3.7657564$ or $x=5.3470455$ (A3) for correct values

$$A=\int_{1.8871981}^{3.7657564}(f(x)-g(x))\mathrm{d}x+\int_{3.7657564}^{5.3470455}(g(x)-f(x))\mathrm{d}x$$ M1A1

$$=\int_{1.8871981}^{3.7657564}\left((x^3-11x^2+38x-40)-\left(\frac{2}{3}x-2\right)\right)\mathrm{d}x+\int_{3.7657564}^{5.3470455}\left(\left(\frac{2}{3}x-2\right)-(x^3-11x^2+38x-40)\right)\mathrm{d}x$$ A1

$=2.784974546+1.758993035$

$=4.543967581$

$=4.54$ A1 N6

[8]

(b) $\int_2^6 f'(x)h(x)\mathrm{d}x=16-\int_2^6 f(x)h'(x)\mathrm{d}x$ (M1) for setting equation

$\int_2^6 f'(x)h(x)\mathrm{d}x+\int_2^6 f(x)h'(x)\mathrm{d}x=16$

$\int_2^6 (f'(x)h(x)+f(x)h'(x))\mathrm{d}x=16$

$\int_2^6 \frac{\mathrm{d}}{\mathrm{d}x}(f(x)h(x))\mathrm{d}x=16$ (M1) for valid approach

$\int_2^6 k'(x)\mathrm{d}x=16$

$k(6)-k(2)=16$ R1

$f(6)h(6)-f(2)h(2)=16$ (M1) for valid approach

$(8)(h(6))-(0)(h(2))=16$ (A1) for substitution

$8h(6)=16$

$h(6)=2$ A1 N4

[6]

4. (a) $f(x)=g(x)$ (M1) for setting equation

$-0.5x^3+4x^2-8.5x+5=2-0.5x$

$-0.5x^3+4x^2-8x+3=0$

$x=0.4858631$, $x=2.4280067$ or $x=5.0861302$ (A3) for correct values

A

$=\int_{0.4858631}^{2.4280067}(g(x)-f(x))\mathrm{d}x$

$+\int_{2.4280067}^{5.0861302}(f(x)-g(x))\mathrm{d}x$ (M1) for valid approach

$=\int_{0.4858631}^{2.4280067}((2-0.5x)-(-0.5x^3+4x^2-8.5x+5))\mathrm{d}x$

$+\int_{2.4280067}^{5.0861302}((-0.5x^3+4x^2-8.5x+5)-(2-0.5x))\mathrm{d}x$ A2

$=2.215507111+5.119788446$

$=7.335295557$

$=7.34$ A1 N6

[8]

(b) $\int_1^4 h'(f(x))\cdot f'(x)\mathrm{d}x=10$

$\int_1^4 \frac{\mathrm{d}}{\mathrm{d}x}(h(f(x)))\mathrm{d}x=10$ M1A1

$\int_1^4 k'(x)\mathrm{d}x=10$

$k(4)-k(1)=10$ R1

$h(f(4))-h(f(1))=10$ (M1) for valid approach

$h(3)-h(0)=10$ A1

$h(3)-7=10$ (A1) for correct equation

$h(3)=17$ A1 N4

[7]

Chapter 17 Solution

Exercise 73

1. (a) The mean

$=\frac{150}{15}$ (A1) for correct formula

$=10$ A1 N2

[2]

(b) (i) 30 A1 N1

(ii) The new variance

$=(3^2)(8)$ (M1) for valid approach

$=72$ A1 N2

[3]

2. (a) The sum of the items

$=(12)(9)$ (A1) for correct formula

$=108$ A1 N2

[2]

(b) (i) 19 A1 N1

(ii) The new standard deviation

$=\sqrt{2.25}$ (M1) for valid approach

$=1.5$ A1 N2

[3]

3. (a) The upper quartile

$=\frac{20+22}{2}$ (M1) for valid approach

$=21$ A1 N2

[2]

(b) (i) 40 A1 N1

(ii) The new inter-quartile range

$=4(21-10)$ (M1) for valid approach

$=44$ A1 N2

[3]

4. (a) The lower quartile

$=\frac{8+12}{2}$ (M1) for valid approach

$=10$ A1 N2

[2]

(b) (i) 19 A1 N1

(ii) The new upper quartile

$=10+5+19$ (M1) for valid approach

$=34$ A1 N2

[3]

Exercise 74

1.	(a)	$a = 3$	A1	N1
		$b = 14$	A1	N1
				[2]
	(b)	$p > 14 + 1.5(6)$	(M2) for definition of outliers	
		$p > 23$	(A1) for correct value	
		Thus, the least value of p is 24.	A1	N3
				[4]
2.	(a)	$a = 63$	A1	N1
		$b = 73$	A1	N1
				[2]
	(b)	$k > 73 + 1.5(10)$	(M2) for definition of outliers	
		$k > 88$	(A1) for correct value	
		Thus, the least value of k is 89.	A1	N3
				[4]
3.	(a)	(i) 34	A1	N1
		(ii) 24	A1	N1
		(iii) 12	A1	N1
				[3]
	(b)	As the median is 34, the number of data less than 34 is the same as that of greater than 34.	R1	
		$\therefore 2 + 4 = q + 1$	(A1) for correct equation	
		$q = 5$	A1	N3
				[3]
4.	(a)	(i) 5	A1	N1
		(ii) 8	A1	N1
		(iii) 6	A1	N1
				[3]
	(b)	As the median is 5, the number of data less than 5 is the same as that of greater than 5.	R1	
		$\therefore 1 + r = 5 + 3 + 2$	(A1) for correct equation	
		$r = 9$	A1	N3
				[3]

Exercise 75

1.	(a)	(i)	\$7.5	A2	N2
		(ii)	20	A1	N1
					[3]
	(b)	(i)	The number of learning points $=(5)(15)$ $=75$	A1	N1
		(ii)	The number of learning points $=(5)(15)+(10)(20-15)$ $=125$	(M1)(A1) for correct formula A1	 N3
					[4]
	(c)		The amount raised $=\frac{62.5}{5}$ $=\$12.5$ Thus, the number of students $=120-50$ $=70$	(M1) for valid approach (A1) for correct formula A1	 N3
					[3]
	(d)		The number of students awarded not more than k learning points $=120-80$ $=40$ k $=(5)(10)$ $=50$	(M1) for valid approach (A1) for correct value (A1) for correct formula A1	 N3
					[4]
	(e)		Simple random sampling	A1	N1
					[1]

2. (a) (i) 1.5 cm A2 N2

(ii) 20 A1 N1

(iii) The percentage of fish

$= \frac{100-20}{200} \times 100\%$ (M1) for valid approach

$= 40\%$ A1 N2

(iv) The number of fish not longer than k cm

$= 200 \times (1-90\%)$ (M1) for valid approach

$= 20$ (A1) for correct value

$\therefore k = 1$ A1 N3

[8]

(b) The price

$= (20)(4.5)$ (M1) for valid approach

$= \$90$ A1 N2

[2]

(c) The number of fish

$= 200 \times (1-10\%)$ (M1) for valid approach

$= 180$

180 fish are not longer than 4 cm.

Thus, 20 fish are longer than 4 cm. (A1) for correct value

r

$= (20)(4)$ (A1) for correct formula

$= 80$ A1 N3

[4]

3.	(a)	25 minutes	A2 N2	
				[2]
	(b)	15 minutes	A2 N2	
				[2]
	(c)	The number of students whose travelling time is within 5 minutes of the median		
		= The number of students whose travelling time is between 20 minutes and 30 minutes	(M1) for valid approach	
		$=120-60$	A1	
		$=60$	A1 N3	
				[3]
	(d)	The number of students spent not more than k minutes to travel to school		
		$=160-160\times\frac{1}{16}$	(M1) for valid approach	
		$=160-10$		
		$=150$	(A1) for correct value	
		$\therefore k=40$	A1 N3	
				[3]
	(e)	r		
		$=30+(1.5)(15)$	(M1)(A1) for correct formula	
		$=52.5$	A1 N3	
				[3]
	(f)	Systematic sampling	A1 N1	
				[1]

4. (a) 35 minutes — A2 N2

[2]

(b) 10 minutes — A2 N2

[2]

(c) The number of secretaries whose time for presentation is within 5 minutes of the upper quartile
$=$ The number of secretaries whose time for presentation is between 35 minutes and 45 minutes — (M1) for valid approach
$=70-40$ — A1
$=30$ — A1 N3

[3]

(d) The number of secretaries spent not more than k minutes to complete a presentation
$=80(1-87.5\%)$ — (M1) for valid approach
$=100$ — (A1) for correct value
$\therefore k=25$ — A1 N3

[3]

(e) r
$=40+(1.5)(10)$ — (M1)(A1) for correct formula
$=55$ — A1 N3

[3]

(f) The probability
$=\dfrac{80-75}{80}$ — (M1) for valid approach
$=\dfrac{1}{16}$ — A1 N2

[2]

Exercise 76

1. (a) (i) p

$= 14 + 7$

$= 21$ A1 N1

(ii) q

$= 39 - 21$ (M1) for valid approach

$= 18$ A1 N2

[3]

(b) The mean number of notebooks

$= \frac{(1)(14)+(2)(7)+(3)(18)+(4)(10)+(5)(1)}{50}$ (M1) for valid approach

$= 2.54$ A1 N2

[2]

(c) 1.15 A1 N1

[1]

2. (a) (i) p

$= 53 + 37$

$= 90$ A1 N1

(ii) q

$= 165 - 115$ (M1) for valid approach

$= 50$ A1 N2

[3]

(b) The mean number of sit-ups

$= \frac{(22)(32)+(23)(21)+(24)(37)+(25)(25)+(26)(50)+(27)(15)}{180}$ (M1) for valid approach

$= 24.47222222$

$= 24.5$ A1 N2

[2]

(c) 2.60 A1 N1

[1]

3. (a) $\dfrac{(1)(2)+(2)(4)+(3)(6)+(4)(16)+5p+(6)(10)}{2+4+6+16+p+10}=4.24$ M1A1

$\dfrac{5p+152}{p+38}=4.24$ (A1) for correct formula

$5p+152=4.24p+161.12$

$0.76p=9.12$

$p=12$ A1 N2

[4]

(b) q

$=12+28+10$ (M1) for valid approach

$=50$ A1 N2

[2]

(c) Discrete A1 N1

[1]

4. (a) $\dfrac{(7)(5)+(12)(3)+(17)(6)+(22)(5)+27p}{5+3+6+5+p}=17.8$ M1A1

$\dfrac{27p+283}{p+19}=17.8$ (A1) for correct formula

$27p+283=17.8p+338.2$

$9.2p=55.2$

$p=6$ A1 N2

[4]

(b) The upper quartile

$=\dfrac{19\text{th}+20\text{th}}{2}$ (M1) for valid approach

$=\dfrac{22+27}{2}$

$=24.5$ A1 N2

[2]

Chapter 18 Solution

Exercise 77

1. (a) (i) The required probability

$= \frac{3+2+4+3}{20}$ (A1) for correct formula

$= \frac{3}{5}$ A1 N2

(ii) The required probability

$= \frac{3+5}{3+3+5}$ (A1) for correct formula

$= \frac{8}{11}$ A1 N2

[4]

(b) The required probability

$= \left(\frac{3+2+3+3}{20}\right)\left(\frac{3+2+3+3-1}{20-1}\right)$ A2

$= \left(\frac{11}{20}\right)\left(\frac{10}{19}\right)$

$= \frac{11}{38}$ A1 N1

[3]

2. (a) (i) The required probability

$= \frac{2+10+3+5+10}{50}$ (A1) for correct formula

$= \frac{3}{5}$ A1 N2

(ii) The required probability

$= \frac{3+5+10}{10+3+5+10}$ (A1) for correct formula

$= \frac{9}{14}$ A1 N2

[4]

(b) The required probability

$= \left(\frac{5+10}{50}\right)\left(\frac{5+10-1}{50-1}\right)$ A2

$= \left(\frac{15}{50}\right)\left(\frac{14}{49}\right)$

$= \frac{3}{35}$ A1 N1

[3]

3. (a) (i) The required probability

$= \frac{2+1+5+3+4+2+1}{25}$ (A1) for correct formula

$= \frac{18}{25}$ A1 N2

(ii) The required probability

$= \frac{5}{1+5+2}$ (A1) for correct formula

$= \frac{5}{8}$ A1 N2

[4]

(b) The required probability

$= \left(\frac{4+2+1}{25}\right)\left(\frac{4+2+1-1}{25-1}\right)$ A2

$= \left(\frac{7}{25}\right)\left(\frac{6}{24}\right)$

$= \frac{7}{100}$ A1 N1

[3]

4. (a)

$$\frac{5+15+a}{100}=\frac{6}{25}$$ (M1) for setting equation

$20+a=24$

$a=4$ A1

$5+15+4+\cdots+15+b=100$

$b=6$ A1 N3

[3]

(b) The required probability

$$=\frac{15+4+10+10+15+6}{15+4+5+5+10+10+15+6}$$ (A1) for correct formula

$$=\frac{6}{7}$$ A1 N2

[2]

(c) The required probability

$$=\left(\frac{6}{100}\right)\left(\frac{6-1}{100-1}\right)$$ A2

$$=\left(\frac{6}{100}\right)\left(\frac{5}{99}\right)$$

$$=\frac{1}{330}$$ A1 N1

[3]

Exercise 78

1.	(a)	(i)	$a+9=13$ $a=4$	(M1) for valid approach A1 N2
		(ii)	$21+4+b=30$ $b=5$	(M1) for valid approach A1 N2
				[4]
	(b)	The required probability $=\frac{4}{30}$ $=\frac{2}{15}$		(M1) for valid approach A1 N2
				[2]
2.	(a)	(i)	$17+15-h+10=40$ $h=2$	(M1) for valid approach A1 N2
		(ii)	$2+k=15$ $k=13$	(M1) for valid approach A1 N2
				[4]
	(b)	The required probability $=\frac{2}{40}$ $=\frac{1}{20}$		(M1) for valid approach A1 N2
				[2]
3.	(a)	(i)	$p=0.4$	A1 N1
		(ii)	$0.4+q=0.6$ $q=0.2$	(M1) for valid approach A1 N2
				[3]
	(b)	$P(A\cup B)=P(A)+P(B)-P(A\cap B)$ $0.9=P(A)+0.6-0.4$ $P(A)=0.7$		(M1) for valid approach (A1) for substitution A1 N2
				[3]

4. (a) (i) $a = 0.3$ A1 N1

(ii) $0.3 + b = 1 - 0.6$ (M1) for valid approach

$b = 0.1$ A1 N2

[3]

(b) $\mathrm{P}(A \cup B) = \mathrm{P}(A) + \mathrm{P}(B) - \mathrm{P}(A \cap B)$ (M1) for valid approach

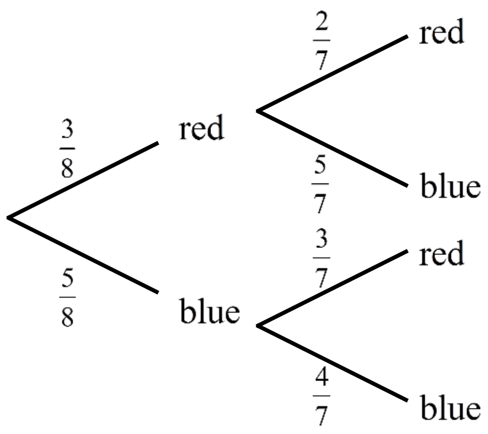
$1 - 0.3 = 0.6 + 0.2 - \mathrm{P}(A \cap B)$ (A1) for substitution

$\mathrm{P}(A \cap B) = 0.1$ A1 N2

[3]

Exercise 79

1. (a)



A3 N3

[3]

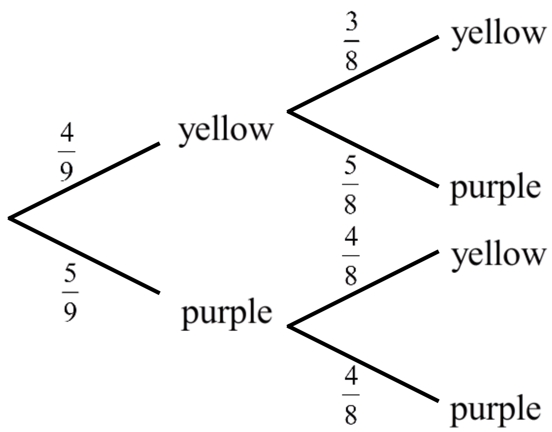
(b) The required probability

$= \left(\frac{3}{8}\right)\left(\frac{5}{7}\right)+\left(\frac{5}{8}\right)\left(\frac{3}{7}\right)$ (M1)(A1) for correct formula

$= \frac{15}{28}$ A1 N2

[3]

2. (a)



A3 N3

[3]

(b) The required probability

$= \left(\frac{4}{9}\right)\left(\frac{5}{8}\right)+\frac{5}{9}$ (M1)(A1) for correct formula

$= \frac{5}{6}$ A1 N2

[3]

3. (a) $x = \frac{5}{8}$ A1 N1

[1]

(b) $\text{P}(B)$

$= \left(\frac{3}{8}\right)\left(\frac{1}{5}\right) + \left(\frac{5}{8}\right)\left(\frac{2}{5}\right)$ (M1)(A1) for correct formula

$= \frac{13}{40}$ A1 N2

[3]

(c) $\text{P}(A \mid B) = \frac{\text{P}(A \cap B)}{\text{P}(B)}$ (M1) for valid approach

$\text{P}(A \mid B) = \frac{\left(\frac{3}{8}\right)\left(\frac{1}{5}\right)}{\frac{13}{40}}$ (A1) for substitution

$\text{P}(A \mid B) = \frac{3}{13}$ A1 N2

[3]

4. (a) $\text{P}(B \mid A') = \frac{3}{5}$ A1 N1

[1]

(b) $\text{P}(B)$

$= \left(\frac{1}{3}\right)\left(\frac{4}{5}\right) + \left(\frac{2}{3}\right)\left(\frac{3}{5}\right)$ (M1)(A1) for correct formula

$= \frac{2}{3}$ A1 N2

[3]

(c) $\text{P}(A' \mid B) = \frac{\text{P}(A' \cap B)}{\text{P}(B)}$ (M1) for valid approach

$\text{P}(A' \mid B) = \frac{\left(\frac{2}{3}\right)\left(\frac{3}{5}\right)}{\frac{2}{3}}$ (A1) for substitution

$\text{P}(A' \mid B) = \frac{3}{5}$ A1 N2

[3]

Exercise 80

1.	(a)	$P(A)$	
		$= P(A \cap B) + P(A \cap B')$	(M1) for valid approach
		$= 0.08 + 0.12$	
		$= 0.2$	A1 N2
			[2]
	(b)	$P(A \cap B) = P(A) \times P(B)$	(M1) for valid approach
		$0.08 = 0.2 \times P(B)$	
		$P(B) = 0.4$	(A1) for correct value
		$P(A \cup B) = P(A) + P(B) - P(A \cap B)$	(A1) for correct formula
		$P(A \cup B) = 0.2 + 0.4 - 0.08$	
		$P(A \cup B) = 0.52$	A1 N3
			[4]
2.	(a)	$P(B) = P(A \cap B) + P(A' \cap B)$	(M1) for valid approach
		$0.3 = P(A \cap B) + 0.15$	
		$P(A \cap B) = 0.15$	A1 N2
			[2]
	(b)	$P(A \cap B) = P(A) \times P(B)$	(M1) for valid approach
		$0.15 = P(A) \times 0.3$	
		$P(A) = 0.5$	(A1) for correct value
		$P(A \cup B) = P(A) + P(B) - P(A \cap B)$	(A1) for correct formula
		$P(A \cup B) = 0.5 + 0.3 - 0.15$	
		$P(A \cup B) = 0.65$	A1 N3
			[4]
3.	$P(A) = P(A \cap B) + P(A \cap B')$		(M1) for valid approach
	$0.4 = P(A \cap B) + 0.28$		
	$P(A \cap B) = 0.12$		A1
	$P(A \cap B) = P(A) \times P(B)$		(M1) for valid approach
	$0.12 = 0.4 \times P(B)$		
	$P(B) = 0.3$		(A1) for correct value
	$P(A \cup B) = P(A) + P(B) - P(A \cap B)$		(A1) for correct formula
	$P(A \cup B) = 0.4 + 0.3 - 0.12$		
	$P(A \cup B) = 0.58$		A1 N4
			[6]

4. $P(A \cap B) = P(A) \times P(B)$ (M1) for valid approach

$0.21 = 0.7P(B)$

$P(B) = 0.3$ A1

$P(A \cup B) = P(A) + P(B) - P(A \cap B)$ (A1) for correct formula

$P(A \cup B) = 0.7 + 0.3 - 0.21$

$P(A \cup B) = 0.79$ A1

$P(A' \cap B')$

$= 1 - P(A \cup B)$ (M1)(A1) for correct formula

$= 1 - 0.79$

$= 0.21$ A1 N4

[7]

Exercise 81

1. (a) $P(C \cap D)$

$= P(C) \times P(D)$

$= 2k^2 \times 3k^2$ (A1) for substitution

$= 6k^4$ A1 N2

[2]

(b) $6k^4 = 0.0096$ (A1) for correct equation

$k^4 = 0.0016$

$k = 0.2$ A1 N2

[2]

(c) $P(C) = P(C \cap D) + P(C \cap D')$

$2(0.2)^2 = 6(0.2)^4 + P(C \cap D')$ (A1) for substitution

$P(C \cap D') = 0.0704$

$P(D' | C)$

$= \dfrac{P(D' \cap C)}{P(C)}$

$= \dfrac{0.0704}{2(0.2)^2}$ (A1) for substitution

$= 0.88$ A1 N2

[3]

2. (a) $P(E \cap F)$

$= P(E) \times P(F)$

$= 4k^3 \times k$ (A1) for substitution

$= 4k^4$ A1 N2

[2]

(b) $4k^4 = \dfrac{1}{2500}$ (A1) for correct equation

$k^4 = \dfrac{1}{10000}$

$k = \dfrac{1}{10}$ A1 N2

[2]

(c) $P(E \cup F)$

$= P(E) + P(F) - P(E \cap F)$

$= 4\left(\dfrac{1}{10}\right)^3 + \dfrac{1}{10} - 4\left(\dfrac{1}{10}\right)^4$ (A1) for substitution

$= \dfrac{259}{2500}$ A1 N2

[2]

3. (a) $P(A \cap B)$

$= P(A) \times P(B)$

$= 2k \times 1.5(2k)$ (A1) for substitution

$= 6k^2$ A1

$P(A \cup B) = P(A) + P(B) - P(A \cap B)$

$6k - 1 = 2k + 1.5(2k) - 6k^2$ (A1) for correct equation

$6k - 1 = 5k - 6k^2$

$6k^2 + k - 1 = 0$ (M1) for valid approach

$(3k - 1)(2k + 1) = 0$

$k = \frac{1}{3}$ or $k = -\frac{1}{2}$ (*Rejected*) A1 N3

[5]

(b) $P(B \mid A)$

$= \frac{P(A \cap B)}{P(A)}$

$= \frac{6\left(\frac{1}{3}\right)^2}{2\left(\frac{1}{3}\right)}$ (A1) for substitution

$= 1$ A1 N2

[2]

4. $P(A \cap B) = P(A) \times P(B)$ R1

$P(A \cap B) = P(A) \times 3P(A)$

$P(A \cap B) = 3P(A)^2$ (A1) for correct formula

$P(A \cup B) = P(A) + P(B) - P(A \cap B)$ (M1) for valid approach

$0.93 = P(A) + 3P(A) - 3P(A)^2$ A1

$3P(A)^2 - 4P(A) + 0.93 = 0$

$300P(A)^2 - 400P(A) + 93 = 0$

$(30P(A) - 31)(10P(A) - 3) = 0$

$P(A) = \frac{31}{30}$ (*Rejected*) or $P(A) = \frac{3}{10}$ A2

$\therefore P(B)$

$= 3\left(\frac{3}{10}\right)$

$= \frac{9}{10}$ A1 N6

[7]

Exercise 82

1. (a) $P(A\cup B)=P(A)+P(B)-P(A\cap B)$ (M1) for valid approach

$1=0.4+0.65-P(A\cap B)$

$P(A\cap B)=0.05$ A1 N2

[2]

(b) $P(A\cap B)+P(A'\cap B)=P(B)$ (M1) for valid approach

$0.05+P(A'\cap B)=0.65$

$P(A'\cap B)=0.6$ A1 N2

[2]

(c) (i) $P(A\cap C)$

$=P(A|C)\times P(C)$ (M1) for valid approach

$=0.78\times 0.7$ (A1) for substitution

$=0.546$ A1 N3

(ii) $P(A\cap C)$

$=0.546$

$\neq 0$ R1

Thus, A and C are **not** mutually exclusive. AG N0

Valid reasoning

(iii) $P(A)\times P(C)$

$=0.4\times 0.7$

$=0.28$ A1

$\neq 0.546$ R1

$=P(A\cap C)$

Thus, A and C are **not** independent. AG N0

[6]

(d) $P(A\cap C)+P(A'\cap C)=P(C)$ (M1) for valid approach

$0.546+P(A'\cap C)=0.7$ (A1) for substitution

$P(A'\cap C)=0.154$ (A1) for correct value

$P(A)+P(A')=1$

$0.4+P(A')=1$

$P(A')=0.6$ A1

$P(C|A')$

$=\dfrac{P(C\cap A')}{P(A')}$ (M1) for valid approach

$=\dfrac{0.154}{0.6}$

$=0.256666666$

$=0.257$ A1 N3

[6]

2. (a) $\text{P}(A \cup T) = \text{P}(A) + \text{P}(T) - \text{P}(A \cap T)$ (M1) for valid approach

$1 = 0.55 + 0.7 - \text{P}(A \cap T)$

$\text{P}(A \cap T) = 0.25$

Thus, the required percentage is 25%. A1 N2

[2]

(b) $\text{P}(A \cup T) - \text{P}(A \cap T)$ (M1) for valid approach

$= 1 - 0.25$

$= 0.75$

Thus, the required percentage is 75%. A1 N2

[2]

(c) (i) $\text{P}(M \cap A)$

$= \text{P}(A \mid M) \times \text{P}(M)$ (M1) for valid approach

$= 0.72 \times 0.63$ (A1) for substitution

$= 0.4536$ A1 N3

(ii) $\text{P}(M) \times \text{P}(A)$

$= 0.63 \times 0.55$

$= 0.3465$ A1

$\neq 0.4536$ R1

$= \text{P}(M \cap A)$

Thus, M and A are **not** independent. AG N0

[5]

(d) $\text{P}(M \cap A) + \text{P}(M' \cap A) = \text{P}(A)$ (M1) for valid approach

$0.4536 + \text{P}(M' \cap A) = 0.55$ (A1) for substitution

$\text{P}(M' \cap A) = 0.0964$ (A1) for correct value

$\text{P}(M) + \text{P}(M') = 1$

$0.63 + \text{P}(M') = 1$

$\text{P}(M') = 0.37$ A1

$\text{P}(A \mid M')$

$= \dfrac{\text{P}(A \cap M')}{\text{P}(M')}$ (M1) for valid approach

$= \dfrac{0.0964}{0.37}$

$= 0.26054054$

$= 0.261$ A1 N3

[6]

3. (a) $P(F \cup R) = P(F) + P(R) - P(F \cap R)$ (M1) for valid approach

$1 = 0.85 + 0.45 - P(F \cap R)$

$P(F \cap R) = 0.3$

Thus, the required percentage is 30%. A1 N2

[2]

(b) $P(F \cup R) - P(F \cap R)$ (M1) for valid approach

$= 1 - 0.3$

$= 0.7$

Thus, the required percentage is 70%. A1 N2

[2]

(c) (i) $P(R \mid F)$

$= \dfrac{P(R \cap F)}{P(F)}$

$= \dfrac{0.3}{0.85}$ (M1) for substitution

$= 0.352941176$

$= 0.353$ A1 N2

(ii) $P(R \mid (F \cap R)')$

$= \dfrac{P(R \cap (F \cap R)')}{P((F \cap R)')}$

$= \dfrac{P(F' \cap R)}{1 - P(F \cap R)}$

$= \dfrac{1 - 0.85}{1 - 0.3}$ (M1) for substitution

$= 0.214285714$

$= 0.214$ A1 N2

[4]

(d) (i) $P(F \cap T)$

$= P(F \mid T) \times P(T)$ (M1) for valid approach

$= 0.9 \times 0.6$

$= 0.54$ A1

$P(F \cap T) \neq 0$ R1

Thus, F and T are **not** mutually exclusive. AG N0

(ii) $P(F) \times P(T)$

$= 0.85 \times 0.6$

$= 0.51$ A1

$\neq 0.54$ R1

Thus, F and T are **not** independent. AG N0

(iii) $P(F \cap T) + P(F \cap T') = P(F)$

$0.54 + P(F \cap T') = 0.85$ (M1) for substitution

$P(F \cap T') = 0.31$

Thus, the required percentage is 31%. A1 N2

[7]

4. (a) $P(Q \cup T) = P(Q) + P(T) - P(Q \cap T)$ (M1) for valid approach

$1 - 0.25 = 0.35 + 0.5 - P(Q \cap T)$

$P(Q \cap T) = 0.1$

Thus, the required percentage is 10%. A1 N2

[2]

(b) $P(Q \cap T) + P(Q' \cap T) = P(T)$ (M1) for valid approach

$0.1 + P(Q' \cap T) = 0.5$

$P(Q' \cap T) = 0.4$

Thus, the required percentage is 40%. A1 N2

[2]

(c) (i) $P(Q' \cap T \mid Q \cup T)$

$= \dfrac{P(Q' \cap T)}{P(Q \cup T)}$

$= \dfrac{0.4}{1 - 0.25}$ (M1) for substitution

$= 0.533333333$

$= 0.533$ A1 N2

(ii) $P(Q \mid T)$

$= \dfrac{P(Q \cap T)}{P(T)}$

$= \dfrac{0.1}{0.5}$ (M1) for substitution

$= 0.2$ A1 N2

[4]

(d) (i) $P(T \cap G)$

$= P(T \mid G) \times P(G)$ (M1) for valid approach

$= 0.95 \times 0.4$

$= 0.38$ A1

$P(T \cap G) \neq 0$ R1

Thus, T and G are **not** mutually exclusive. AG N0

(ii) $P(T) \times P(G)$

$= 0.5 \times 0.4$

$= 0.2$ A1

$\neq 0.38$ R1

$= P(T \cap G)$

Thus, T and G are **not** independent. AG N0

(iii) $P(T \cap G) + P(T \cap G') = P(T)$

$0.38 + P(T \cap G') = 0.5$ (M1) for substitution

$P(T \cap G') = 0.12$

Thus, the required percentage is 12%. A1 N2

[7]

Chapter 19 Solution

Exercise 83

1.	$P(X=1)+P(X=2)+P(X=3)+P(X=4)=1$	(M1) for sum of probabilities
	$0.2+0.3+a+b=1$	A1
	$a+b=0.5$	
	$E(X)=2.62$	
	$0.2(1)+0.3(2)+3a+4b=2.62$	A1
	$\therefore 0.2+0.6+3a+4(0.5-a)=2.62$	(M1) for substitution
	$2.8-a=2.62$	(A1) for simplification
	$a=0.18$	A1 N4
		[6]
2.	$P(X=20)+P(X=30)+P(X=40)+P(X=50)=1$	(M1) for sum of probabilities
	$0.1+0.1+a+b=1$	A1
	$a+b=0.8$	
	$E(X)=33$	
	$0.1(20)+30a+40b+0.1(50)=33$	A1
	$\therefore 30a+40(0.8-a)+7=33$	(M1) for substitution
	$-10a=-6$	(A1) for simplification
	$a=0.6$	A1
	$b=0.8-0.6$	
	$b=0.2$	A1 N4
		[7]
3.	$P(X<15)=0.5$	
	$P(X=0)+P(X=10)=0.5$	(M1) for sum of probabilities
	$0.1+a=0.5$	
	$a=0.4$	A1
	$P(X=0)+P(X=10)+P(X=20)+P(X=30)=1$	
	$0.1+0.4+b+c=1$	(A1) for substitution
	$b+c=0.5$	
	$E(X)=16$	
	$0.1(0)+10a+20b+30c=16$	A1
	$\therefore 4+20b+30(0.5-b)=16$	(M1) for substitution
	$-10b=-3$	
	$b=0.3$	A1
	$c=0.5-0.3$	
	$c=0.2$	A1 N4
		[7]

4. $\mathrm{P}(2 < X < 7) = 0.3$

$\mathrm{P}(X = 3) + \mathrm{P}(X = 6) = 0.3$ (M1) for sum of probabilities

$a + b = 0.3$

$b = 0.3 - a$ A1

$\mathrm{P}(X = 0) + \mathrm{P}(X = 3) + \mathrm{P}(X = 6) + \mathrm{P}(X = 9) = 1$

$0.4 + a + b + c = 1$ (A1) for substitution

$a + b + c = 0.6$

$\therefore 0.3 + c = 0.6$

$c = 0.3$ (A1) for correct value

$\mathrm{E}(X) = 4.2$

$0.4(0) + 3a + 6(0.3 - a) + 9(0.3) = 4.2$ (M1) for substitution

$-3a + 4.5 = 4.2$

$a = 0.1$ A1

$b = 0.3 - 0.1$

$b = 0.2$ A1 N4

[7]

Exercise 84

1. (a) $P(X=0)+P(X=1)+P(X=2)+P(X=3)=1$ (M1) for sum of probabilities

$9k+k+0.1+0.4=1$

$10k=0.5$ (A1) for simplification

$k=0.05$ A1 N2

[3]

(b) $E(X)$

$=9k(0)+k+0.1(2)+0.4(3)$ (A1) for correct formula

$=0+0.05+0.2+1.2$ (A1) for substitution

$=1.45$ A1 N2

[3]

2. (a) $P(X=0)+P(X=20)+P(X=40)+P(X=60)=1$ (M1) for sum of probabilities

$\frac{1}{10}+\frac{1}{5}+\frac{2}{5}+k=1$

$\frac{7}{10}+k=1$ (A1) for simplification

$k=\frac{3}{10}$ A1 N2

[3]

(b) $E(X)$

$=\frac{1}{10}(0)+\frac{1}{5}(20)+\frac{2}{5}(40)+60k$ (A1) for correct formula

$=0+4+16+60\left(\frac{3}{10}\right)$ (A1) for substitution

$=38$ A1 N2

[3]

3. (a) $P(X=1)+P(X=2)+P(X=k)=1$ (M1) for sum of probabilities

$\frac{1}{14}+\frac{4}{14}+\frac{k^2}{14}=1$

$5+k^2=14$ (A1) for simplification

$k^2=9$

$k=3$ A1 N2

[3]

(b) $E(X)$

$=\frac{1}{14}(1)+\frac{4}{14}(14)+\frac{k^2}{14}(k)$ (A1) for correct formula

$=\frac{1}{14}+4+\frac{3^3}{14}$ (A1) for substitution

$=6$ A1 N2

[3]

4. (a) $P(X=k)+P(X=k+1)+P(X=k+2)+P(X=8)=1$ (M1) for sum of probabilities

$\frac{k}{2}+\frac{1}{8}+\frac{k}{4}+\frac{1}{8}=1$

$\frac{3k}{4}+\frac{1}{4}=1$ (A1) for simplification

$3k+1=4$

$k=1$ A1 N2

[3]

(b) $E(X)$

$=\frac{k}{2}(k)+\frac{1}{8}(k+1)+\frac{k}{4}(k+2)+\frac{1}{8}(8)$ (A1) for correct formula

$=\frac{1}{2}+\frac{2}{8}+\frac{3}{4}+1$ (A1) for substitution

$=\frac{5}{2}$ A1 N2

[3]

Exercise 85

1. (a) (i) $P(F \cap S)$

$= (0.6)(0.6)$

$= 0.36$ A1 N1

(ii) $P(S)$

$= P(F \cap S) + P(F' \cap S)$ (M1) for valid approach

$= 0.36 + (0.4)(0.6)$ (M1) for substitution

$= 0.6$ A1 N2

[4]

(b) (i) The required probability

$= P(F' \cap S')$

$= (0.4)(0.4)$ (A1) for substitution

$= 0.16$ A1 N1

(ii) The required probability

$= P(F \mid S')$ (M1) for valid approach

$= \frac{P(F \cap S')}{P(S')}$

$= \frac{(0.6)(0.4)}{1 - 0.6}$ (A1) for substitution

$= 0.6$ A1 N3

[5]

(c)

X	2	5	8
$P(X = x)$	0.16	0.48	0.36

A3 N3

[3]

(d) The expected value

$= (2)(0.16) + (5)(0.48) + (8)(0.36)$ (M1) for valid approach

$= 5.6$ A1 N2

[2]

2. (a) (i) $P(S \cap L')$

$= (0.2)(0.3)$

$= 0.06$ A1 N1

(ii) $P(L')$

$= P(S \cap L') + P(S' \cap L')$ (M1) for valid approach

$= 0.06 + (0.8)(0.6)$ (M1) for substitution

$= 0.54$ A1 N2

[4]

(b) (i) The required probability

$= P(S' \cap L')$

$= (0.8)(0.6)$ (A1) for substitution

$= 0.48$ A1 N1

(ii) The required probability

$= P(S \mid L)$ (M1) for valid approach

$= \dfrac{P(S \cap L)}{P(L)}$

$= \dfrac{(0.2)(0.7)}{1 - 0.54}$ (A1) for substitution

$= \dfrac{7}{23}$ A1 N3

[5]

(c)

X	0	10	25
$P(X = x)$	0.09	0.42	0.49

A3 N3

[3]

(d) The expected value

$= (0)(0.09) + (10)(0.42) + (25)(0.49)$ (M1) for valid approach

$= 16.45$ A1 N2

[2]

3. (a) (i) $P(T' \cap L')$

$= \left(1-\frac{1}{2}\right)\left(1-\frac{3}{10}\right)$

$= \frac{7}{20}$ A1 N1

(ii) $P(L')$

$= P(T \cap L') + P(T' \cap L')$ (M1) for valid approach

$= \left(\frac{1}{2}\right)\left(1-\frac{9}{10}\right) + \frac{7}{20}$ (M1) for substitution

$= \frac{2}{5}$ A1 N2

[4]

(b) (i) The required probability

$= P(T \cap L')$

$= \left(\frac{1}{2}\right)\left(1-\frac{9}{10}\right)$ (A1) for substitution

$= \frac{1}{20}$ A1 N1

(ii) The required probability

$= P(L' \mid T')$ (M1) for valid approach

$= \frac{P(L' \cap T')}{P(T')}$

$= \frac{\frac{7}{20}}{1-\frac{1}{2}}$ (A1) for substitution

$= \frac{7}{10}$ A1 N3

[5]

(c)

X	0	125	250	375
$P(X = x)$	$\frac{8}{125}$	$\frac{36}{125}$	$\frac{54}{125}$	$\frac{27}{125}$

A3 N3

[3]

(d) The expected expenditure

$= (0)\left(\frac{8}{125}\right) + (125)\left(\frac{36}{125}\right) + (250)\left(\frac{54}{125}\right)$ (M1) for valid approach

$+ (375)\left(\frac{27}{125}\right)$

$= \$225$ A1 N2

[2]

4. (a) (i) $P(R' \cap A)$

$= (1-0.5)(0.4)$

$= 0.2$ A1 N1

(ii) $P(A)$

$= P(R \cap A) + P(R' \cap A)$ (M1) for valid approach

$= (0.5)(0.8) + 0.2$ (M1) for substitution

$= 0.6$ A1 N2

[4]

(b) (i) The required probability

$= P(R \cap A')$

$= (0.5)(1-0.8)$ (A1) for substitution

$= 0.1$ A1 N1

(ii) The required probability

$= P(R \mid A)$ (M1) for valid approach

$= \dfrac{P(R \cap A)}{P(A)}$

$= \dfrac{(0.5)(0.8)}{0.6}$ (A1) for substitution

$= \dfrac{2}{3}$ A1 N3

[5]

(c)

X	0	4	8	12
$P(X = x)$	$\dfrac{27}{125}$	$\dfrac{54}{125}$	$\dfrac{36}{125}$	$\dfrac{8}{125}$

A3 N3

[3]

(d) The expected expenditure

$= (0)\left(\dfrac{27}{125}\right) + (4)\left(\dfrac{54}{125}\right) + (8)\left(\dfrac{36}{125}\right) + (12)\left(\dfrac{8}{125}\right)$ (M1) for valid approach

$= \$4.8$ A1 N2

[2]

Exercise 86

1. (a) (i) There are 4 ways such that $X=5$ (A1) for correct value

$P(X=5)$

$=\frac{4}{36}$

$=\frac{1}{9}$ A1 N2

(ii) There are 6 ways such that $X<5$ (A1) for correct value

$P(X<5)$

$=\frac{6}{36}$

$=\frac{1}{6}$ A1 N2

(iii) $P(X=4 \mid X<6)$

$=\frac{P(X=4\cap X<6)}{P(X<6)}$ M1

$=\frac{P(X=4)}{P(X<6)}$

$=\frac{\frac{3}{36}}{\frac{1}{9}+\frac{1}{6}}$

$=\frac{3}{10}$ A1 N2

[6]

(b) $P(X>5)$

$=1-P(X=5)-P(X<5)$ M1

$=1-\frac{1}{9}-\frac{1}{6}$

$=\frac{13}{18}$ A1

$E(X)=0$ M1

$(3)P(X=5)+(2)P(X<5)+(-k)P(X>5)=0$ M1

$\therefore (3)\left(\frac{1}{9}\right)+(2)\left(\frac{1}{6}\right)+(-k)\left(\frac{13}{18}\right)=0$ A2

$6+6-13k=0$

$k=\frac{12}{13}$ A1 N4

[7]

2. (a) (i) There are 5 ways such that $X = 8$ — (A1) for correct value

$P(X = 8)$

$= \frac{5}{36}$ — A1 N2

(ii) There are 10 ways such that $X > 8$ — (A1) for correct value

$P(X > 8)$

$= \frac{10}{36}$

$= \frac{5}{18}$ — A1 N2

(iii) $P(X > 9 \mid X > 8)$

$= \frac{P(X > 9 \cap X > 8)}{P(X > 8)}$ — M1

$= \frac{P(X > 9)}{P(X > 8)}$

$= \frac{\frac{6}{36}}{\frac{5}{18}}$

$= \frac{3}{5}$ — A1 N2

[6]

(b) $P(X < 8)$

$= 1 - P(X = 8) - P(X > 8)$ — M1

$= 1 - \frac{5}{36} - \frac{5}{18}$

$= \frac{7}{12}$ — A1

$E(X) = 1$ — M1

$(5)P(X = 8) + (k)P(X > 8) + (-1)P(X < 8) = 1$ — M1

$\therefore (5)\left(\frac{5}{36}\right) + (k)\left(\frac{5}{18}\right) + (-1)\left(\frac{7}{12}\right) = 1$ — A2

$25 + 10k - 21 = 36$

$k = 3.2$ — A1 N4

[7]

3. (a) (i) There is only 1 way such that $X = 21$

$\text{P}(X = 21)$

$= \frac{1}{9}$ A1 N1

(ii) There are 5 ways such that $X > 21$

$\text{P}(X > 21)$

$= \frac{5}{9}$ A1 N1

(iii) $\text{P}(30 < X < 33 \mid X > 21)$

$= \frac{\text{P}(30 < X < 33 \cap X > 21)}{\text{P}(X > 21)}$ M1

$= \frac{\text{P}(30 < X < 33)}{\text{P}(X > 21)}$

$= \frac{\frac{2}{9}}{\frac{5}{9}}$ (A1) for substitution

$= \frac{2}{5}$ A1 N2

[5]

(b) $\text{P}(X < 21)$

$= 1 - \text{P}(X = 21) - \text{P}(X > 21)$ M1

$= 1 - \frac{1}{9} - \frac{5}{9}$

$= \frac{1}{3}$ A1

$\text{E}(X) = 8$ M1

$(3k)\text{P}(X = 21) + (k)\text{P}(X > 21) + (0)\text{P}(X < 21) = 8$ M1

$\therefore (3k)\left(\frac{1}{9}\right) + (k)\left(\frac{5}{9}\right) + (0)\left(\frac{1}{3}\right) = 8$ A2

$3k + 5k = 72$

$k = 9$ A1 N4

[7]

4. (a) (i) There is only 1 way such that $X = 33$

$P(X = 33)$

$= \frac{1}{9}$ A1 N1

(ii) There are 2 ways such that $X \geq 35$

$P(X \geq 35)$

$= \frac{2}{9}$ A1 N1

(iii) $P(X < 22 \mid X < 33)$

$= \frac{P(X < 22 \cap X < 33)}{P(X < 33)}$ M1

$= \frac{P(X < 22)}{P(X < 33)}$

$= \frac{\frac{5}{9}}{\frac{6}{9}}$ (A1) for substitution

$= \frac{5}{6}$ A1 N2

[5]

(b) $P(X < 33)$

$= 1 - P(X = 33) - P(X > 33)$ M1

$= 1 - \frac{1}{9} - \frac{2}{9}$

$= \frac{2}{3}$ A1

$E(X) = -16$ M1

$(4k)P(X = 33) + (3k)P(X > 33) + (-2k)P(X < 33)$
$= -16$ M1

$\therefore (4k)\left(\frac{1}{9}\right) + (3k)\left(\frac{2}{9}\right) + (-2k)\left(\frac{2}{3}\right) = -16$ A2

$4k + 6k - 12k = -144$

$k = 72$ A1 N4

[7]

Exercise 87

1. (a) $P(X=4)+P(X=8)+P(X=12)=1$ (M1) for sum of probabilities

$10k^2+k+20k^2=1$ (A1) for substitution

$30k^2+k-1=0$

$(6k-1)(5k+1)=0$ A1

$k=\frac{1}{6}$ or $k=-\frac{1}{5}$ (*Rejected*) A1 N2

[4]

(b) $P(X=12 \mid X>6)$

$=\frac{P(X=12 \cap X>6)}{P(X>6)}$

$=\frac{P(X=12)}{P(X>6)}$ (M1) for valid approach

$=\frac{20\left(\frac{1}{6}\right)^2}{20\left(\frac{1}{6}\right)^2+\frac{1}{6}}$ (A1) for substitution

$=\frac{10}{13}$ A1 N2

[3]

2. (a) $P(X=12)+P(X=24)+P(X=30)+P(X=36)=1$ (M1) for sum of probabilities

$k+7k^2+8k^2+k=1$ (A1) for substitution

$15k^2+2k-1=0$

$(5k-1)(3k+1)=0$ A1

$k=\frac{1}{5}$ or $k=-\frac{1}{3}$ (*Rejected*) A1 N2

[4]

(b) $P(X=24 \mid X>20)$

$=\frac{P(X=24 \cap X>20)}{P(X>20)}$

$=\frac{P(X=24)}{P(X>20)}$ (M1) for valid approach

$=\frac{7\left(\frac{1}{5}\right)^2}{7\left(\frac{1}{5}\right)^2+8\left(\frac{1}{5}\right)^2+\frac{1}{5}}$ (A1) for substitution

$=\frac{7}{20}$ A1 N2

[3]

3. (a) $P(X=7)+P(X=14)+P(X=21)$

$+P(X=28)+P(X=35)=1$ (M1) for sum of probabilities

$k+3k+10k^2+6k^2+5k^2=1$ (A1) for substitution

$21k^2+4k-1=0$

$(7k-1)(3k+1)=0$ A1

$k=\frac{1}{7}$ or $k=-\frac{1}{3}$ (*Rejected*) A1 N2

[4]

(b) $P(X<15\,|\,X<25)$

$=\frac{P(X<15\cap X<25)}{P(X<25)}$

$=\frac{P(X<15)}{P(X<25)}$ (M1) for valid approach

$=\frac{\frac{1}{7}+3\left(\frac{1}{7}\right)}{\frac{1}{7}+3\left(\frac{1}{7}\right)+10\left(\frac{1}{7}\right)^2}$ (A1) for substitution

$=\frac{14}{19}$ A1 N2

[3]

4. (a) $P(X=0)+P(X=1)+P(X=2)$

$+P(X=3)+P(X=4)+P(X=5)=1$ (M1) for sum of probabilities

$k^2+k+4k^2+8k^2+4k+k^2=1$ (A1) for substitution

$14k^2+5k-1=0$

$(7k-1)(2k+1)=0$ A1

$k=\frac{1}{7}$ or $k=-\frac{1}{2}$ (*Rejected*) A1 N2

[4]

(b) $P(2<X\le 4\,|\,1<X\le 4)$

$=\frac{P(2<X\le 4\cap 1<X\le 4)}{P(1<X\le 4)}$

$=\frac{P(2<X\le 4)}{P(1<X\le 4)}$ (M1) for valid approach

$=\frac{8\left(\frac{1}{7}\right)^2+4\left(\frac{1}{7}\right)}{4\left(\frac{1}{7}\right)^2+8\left(\frac{1}{7}\right)^2+4\left(\frac{1}{7}\right)}$ (A1) for substitution

$=\frac{9}{10}$ A1 N2

[3]

Chapter 20 Solution

Exercise 88

1. (a) $\text{E}(X)$

$= 80(0.06)$ (A1) for substitution

$= 4.8$ A1 N2

[2]

(b) $X \sim B(80, 0.06)$

$\text{P}(X = 10)$

$= \binom{80}{10}(0.06)^{10}(1-0.06)^{80-10}$ (M1) for valid approach

$= 0.0130924797$

$= 0.0131$ A1 N2

[2]

(c) $\text{P}(X \geq 15)$

$= 1 - \text{P}(X \leq 14)$ (M1) for valid approach

$= 1 - 0.9999251314$ (A1) for correct value

$= 0.0000748686$

$= 0.0000749$ A1 N3

[3]

2. (a) $\text{E}(X)$

$= 135(0.12)$ (A1) for substitution

$= 16.2$ A1 N2

[2]

(b) $X \sim B(135, 0.12)$

$\text{P}(X = 20)$

$= \binom{135}{20}(0.012)^{20}(1-0.12)^{135-20}$ (M1) for valid approach

$= 0.0597993427$

$= 0.0598$ A1 N2

[2]

(c) $\text{P}(X > 16)$

$= 1 - \text{P}(X \leq 16)$ (M1) for valid approach

$= 1 - 0.5449524887$ (A1) for correct value

$= 0.4550475113$

$= 0.455$ A1 N3

[3]

3. (a) $\mathrm{E}(X)$

$= 50(0.02)$ — (A1) for substitution

$= 1$ — A1 N2

[2]

(b) $X \sim B(50, 0.02)$

$\mathrm{P}(X = 9)$

$= \binom{50}{9}(0.02)^9(1-0.02)^{50-9}$ — (M1) for valid approach

$= 0.000000560302$

$= 0.000000560$ — A1 N2

[2]

(c) $\mathrm{P}(X \le 2)$

$= 0.9215722517$ — (M1)A1 for valid approach

$= 0.922$ — A1 N3

[3]

4. (a) $\mathrm{E}(X)$

$= 9(0.69)$ — (A1) for substitution

$= 6.21$ — A1 N2

[2]

(b) $X \sim B(9, 0.69)$

$\mathrm{P}(X = 6)$

$= \binom{9}{6}(0.69)^6(1-0.69)^{9-6}$ — (M1) for valid approach

$= 0.2700591597$

$= 0.270$ — A1 N2

[2]

(c) $\mathrm{P}(X < 3)$

$= \mathrm{P}(X \le 2)$ — (M1) for valid approach

$= 0.005271637$ — (A1) for correct value

$= 0.00527$ — A1 N3

[3]

Exercise 89

1. (a) The required probability

$=\binom{120}{3}p^3(1-p)^{120-3}$

$=\binom{120}{3}p^3(1-p)^{117}$ A2 N2

[2]

(b) $\binom{120}{3}p^3(1-p)^{117}=0.16$ (M1) for setting equation

$\binom{120}{3}p^3(1-p)^{117}-0.16=0$

By considering the graph of

$y=\binom{120}{3}p^3(1-p)^{117}-0.16$, $p=0.0148695$

or $p=0.0388023$.

$\therefore p=0.0149$ or $p=0.0388$ A2 N3

[3]

2. (a) The required probability

$=\binom{5}{4}p^4(1-p)^{5-4}$

$=5p^4(1-p)$ A2 N2

[2]

(b) $5p^4(1-p)=0.3$ (M1) for setting equation

$5p^4(1-p)-0.3=0$

By considering the graph of $y=5p^4(1-p)-0.3$,

$p=0.6381051$ or $p=0.9140419$.

$\therefore p=0.638$ or $p=0.914$ A2 N3

[3]

3. (a) The required probability

$=\binom{10}{9}q^9(1-q)^{10-9}+\binom{10}{10}q^{10}(1-q)^{10-10}$

$=10q^9(1-q)+q^{10}$ A2 N2

[2]

(b) $10q^9(1-q)+q^{10}=0.09$ (M1) for setting equation

$10q^9(1-q)+q^{10}-0.09=0$

By considering the graph of

$y=10q^9(1-q)+q^{10}-0.09$, $q=0.6539559$.

$\therefore q=0.654$ A2 N3

[3]

4. (a) The required probability

$$= \binom{100}{0} q^0 (1-q)^{100-0} + \binom{100}{1} q^1 (1-q)^{100-1}$$

$$= (1-q)^{100} + 100q(1-q)^{99}$$ A2 N2

[2]

(b) $(1-q)^{100} + 100q(1-q)^{99} = 0.03$ (M1) for setting equation

$(1-q)^{100} + 100q(1-q)^{99} - 0.03 = 0$

By considering the graph of

$y = (1-q)^{100} + 100q(1-q)^{99} - 0.03$, $q = 0.0524073$.

$\therefore q = 0.0524$ A2 N3

[3]

Exercise 90

1. (a) The required probability

$= 0.56 \times 0.12 + (1 - 0.56) \times 0.76$ (M1)(A1) for valid approach

$= 0.56 \times 0.12 + 0.44 \times 0.76$ (A1) for simplification

$= 0.4016$ A1 N3

[4]

(b) The required probability

$= \dfrac{0.44 \times 0.76}{0.4016}$ (R1)A1 for correct formula

$= 0.8326693227$

$= 0.833$ A1 N2

[3]

(c) $X \sim B(6, 0.5984)$ (R1) for binomial distribution

$P(X = 4)$

$= 0.3102022951$

$= 0.310$ A1 N2

[2]

(d) The probability that Joyce did not stay at home for all n days

$= 0.4016^n$ (M1) for valid approach

$1 - 0.4016^n > 0.84$ (M1)A1 for setting inequality

$0.4016^n < 0.16$

$0.4016^n - 0.16 < 0$

By considering the graph of $y = 0.4016^n - 0.16$,

$n > 2.0087516$. (A1) for correct value

$\therefore n = 3$ A1 N3

[5]

2. (a) The required probability

$= 0.4 \times 0.2 + (1-0.4) \times 0.3$ (M1)(A1) for valid approach

$= 0.4 \times 0.2 + 0.6 \times 0.3$ (A1) for simplification

$= 0.26$ A1 N3

[4]

(b) The required probability

$= \dfrac{0.6 \times 0.3}{0.26}$ (R1)A1 for correct formula

$= 0.6923076923$

$= 0.692$ A1 N2

[3]

(c) $X \sim B(4, 0.26)$ (R1) for binomial distribution

$P(X = 2)$

$= 0.22210656$

$= 0.222$ A1 N2

[2]

(d) $1 - 0.74^n - n(0.74)^{n-1}(0.26) > 0.75$ (M1)A1 for setting inequality

$0.74^n + 0.26n(0.74)^{n-1} - 0.25 < 0$ (M1) for simplification

By considering the graph of

$y = 0.74^n + 0.26n(0.74)^{n-1} - 0.25$, $n > 9.4689646$. (A1) for correct value

$\therefore n = 10$ A1 N3

[5]

3. (a) The required probability

$= 0.45 \times 0.13 + (1 - 0.45) \times 0.59$ (M1)(A1) for valid approach

$= 0.45 \times 0.13 + 0.55 \times 0.59$ (A1) for simplification

$= 0.383$ A1 N3

[4]

(b) The required probability

$= \dfrac{0.55 \times 0.59}{0.383}$ (R1)A1 for correct formula

$= 0.8472584856$

$= 0.847$ A1 N2

[3]

(c) $X \sim B(7, 0.383)$ (R1) for binomial distribution

$\mathrm{P}(X = 3)$

$= 0.2849738583$

$= 0.285$ A1 N2

[2]

(d) The probability that Lydia caught a fish at most one day

$= (1 - 0.383)^n + n(1 - 0.383)^{n-1}(0.383)$ (M1) for valid approach

$1 - [(1 - 0.383)^n + n(1 - 0.383)^{n-1}(0.383)] > 0.93$ (M1)A1 for setting inequality

$(1 - 0.383)^n + n(1 - 0.383)^{n-1}(0.383) - 0.07 < 0$

By considering the graph of

$y = (1 - 0.383)^n + n(1 - 0.383)^{n-1}(0.383) - 0.07$,

$n > 9.5074803$. (A1) for correct value

$\therefore n = 10$ A1 N3

[5]

4. (a) The required probability

$= p \times 0.3 + (1-p) \times 0.48$ (M1)(A1) for valid approach

$= 0.3p + 0.48 - 0.48p$ (A1) for simplification

$= 0.48 - 0.18p$ A1 N3

[4]

(b) The required probability

$= \dfrac{0.3p}{0.48 - 0.18p}$ R1A2

[3]

(c) $X \sim B(8, 0.3702)$ (R1) for binomial distribution

$P(X = 6)$

$= 0.0285878721$

$= 0.0286$ A1 N2

[2]

(d) The probability that reaching the escape door for at most two trial

$= (1-0.3702)^n + n(1-0.3702)^{n-1}(0.3702)$

$+ \binom{n}{2}(1-0.3702)^{n-2}(0.3702)^2$ (M1) for valid approach

$1 - [0.6298^n + n(0.6298)^{n-1}(0.3702)$

$+ \binom{n}{2}(0.6298)^{n-2}(0.3702)^2] > 0.99$ (M1)A1 for setting inequality

$0.6298^n + n(0.6298)^{n-1}(0.3702)$

$+ \binom{n}{2}(0.6298)^{n-2}(0.3702)^2 - 0.01 < 0$

By considering the graph of

$y = 0.6298^n + n(0.6298)^{n-1}(0.3702)$

$+ \binom{n}{2}(0.6298)^{n-2}(0.3702)^2 - 0.01$,

$n > 19.237508$. (A1) for correct value

$\therefore n = 20$ A1 N3

[5]

Chapter 21 Solution

Exercise 91

1.	(a)	$P(X > 86) = 0.28$	A1 N1	
				[1]
	(b)	$P(80 < X < 86)$		
		$= P(X > 80) - P(X > 86)$	(M1) for valid approach	
		$= 0.5 - 0.28$	(A1) for substitution	
		$= 0.22$	A1 N2	
				[3]
	(c)	$P(74 < X < 80)$		
		$= P(80 < X < 86)$	(M1) for symmetric property	
		$= 0.22$	A1 N2	
				[2]
2.	(a)	$P(X < 270) = 0.15$	A1 N1	
				[1]
	(b)	$P(270 < X < 300)$		
		$= P(X < 300) - P(X < 270)$	(M1) for valid approach	
		$= 0.5 - 0.15$	(A1) for substitution	
		$= 0.35$	A1 N2	
				[3]
	(c)	$P(270 < X < 330)$		
		$= 2 \times P(270 < X < 300)$	(M1) for symmetric property	
		$= 0.7$	A1 N2	
				[2]
3.	(a)	$P(X > 2.7) = 0.07$	A1 N1	
				[1]
	(b)	$P(1.5 < X < 2.7)$		
		$= P(X > 1.5) - P(X > 2.7)$	(M1) for valid approach	
		$= 0.5 - 0.07$	(A1) for substitution	
		$= 0.43$	A1 N2	
				[3]
	(c)	$P(X > 0.3)$		
		$= 2 \times P(1.5 < X < 2.7) + P(X > 2.7)$	(M1) for symmetric property	
		$= 0.93$	A1 N2	
				[2]

4. (a) $\mathrm{P}\left(X > \frac{3}{11}\right)$

$= 1 - \frac{1}{6}$ M1

$= \frac{5}{6}$ A1 N1

[2]

(b) $d - \frac{6}{11} = \frac{6}{11} - \frac{3}{11}$ (M1) for valid approach

$d = \frac{9}{11}$ A1 N2

[2]

(c) $\mathrm{P}\left(\frac{3}{11} < X < d\right)$

$= 1 - 2 \times \mathrm{P}\left(X < \frac{3}{11}\right)$ (M1) for symmetric property

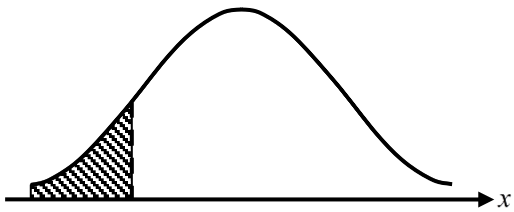
$= \frac{2}{3}$ A1 N2

[2]

Exercise 92

1. (a) For vertical line clearly to the left of the mean A1
For shading to the left of the vertical line A1 N2

[2]



(b) $P(X \le 60) = 0.022750062$ (A1) for correct value
$P(X \le 60) = 0.0228$ A1 N2

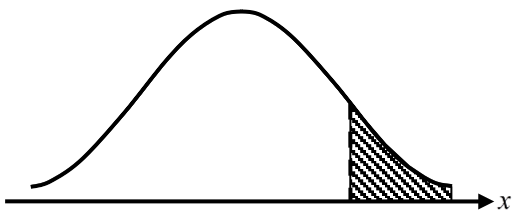
[2]

(c) $c = 61.7961209$
$c = 61.8$ A2 N2

[2]

2. (a) For vertical line clearly to the right of the mean A1
For shading to the right of the vertical line A1 N2

[2]



(b) $P(X \ge 4.83) = 0.2653735838$ (A1) for correct value
$P(X \ge 4.83) = 0.2654$ A1 N2

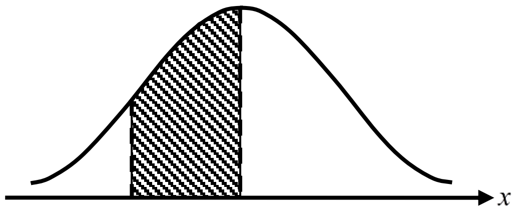
[2]

(c) $c = 4.7613483$
$c = 4.76$ A2 N2

[2]

3. (a) For vertical lines clearly to the left of the mean and at the mean — A1

For shading area bounded by the vertical lines — A1 N2



[2]

(b) $P(23.5 \le X \le 30) = 0.4479187309$ — (A1) for correct value

$P(23.5 \le X \le 30) = 0.4479$ — A1 N2

[2]

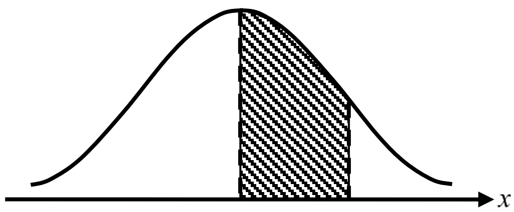
(c) $c = 32.0976017$

$c = 32.1$ — A2 N2

[2]

4. (a) For vertical lines clearly at the mean and to the right of the mean — A1

For shading area bounded by the vertical lines — A1 N2



[2]

(b) $P(162 \le X \le 171) = 0.3697054352$ — (A1) for correct value

$P(162 \le X \le 171) = 0.3697$ — A1 N2

[2]

(c) $c = 158.0331972$

$c = 158$ — A2 N2

[2]

Exercise 93

1. $P(X < 10) = 0.39$

$P\left(X < \frac{10-\mu}{\sigma}\right) = 0.39$ (M1) for standardization

$\frac{10-\mu}{\sigma} = -0.279319035$ (A1) for correct value

$\mu - 0.279319035\sigma = 10 \ldots (1)$ A1

$P(X > 13) = 0.11$

$P\left(X > \frac{13-\mu}{\sigma}\right) = 0.11$

$\frac{13-\mu}{\sigma} = 1.22652812$ (A1) for correct value

$\mu + 1.22652812\sigma = 13 \ldots (2)$ A1

$(1) = (2)$ (M1) for setting equation

Solving, we have $\mu = 10.5564689$ and $\sigma = 1.992234066$.

$\therefore \mu = 10.6$, $\sigma = 1.99$ A2 N4

[8]

2. $P(X > 58) = 0.42$

$P\left(Z > \frac{58-\mu}{\sigma}\right) = 0.42$ (M1) for standardization

$\frac{58-\mu}{\sigma} = 0.2018934725$ (A1) for correct value

$\mu + 0.2018934725\sigma = 58 \ldots (1)$ A1

$P(X > 69) = 0.01$

$P\left(Z > \frac{69-\mu}{\sigma}\right) = 0.01$

$\frac{69-\mu}{\sigma} = 2.326347877$ (A1) for correct value

$\mu + 2.326347877\sigma = 69 \ldots (2)$ A1

$(1) = (2)$ (M1) for setting equation

Solving, we have $\mu = 56.95463445$ and $\sigma = 5.177800651$.

$\therefore \mu = 57.0$, $\sigma = 5.18$ A2 N4

[8]

3. $P(20 < X < \mu) = 0.2$

$P\left(Z < \dfrac{20-\mu}{\sigma}\right) = 0.3$	(M1) for standardization
$\dfrac{20-\mu}{\sigma} = -0.5244005101$	(A1) for correct value
$\mu - 0.5244005101\sigma = 20 \ldots(1)$	A1
$P(\mu < X < 24) = 0.3$	
$P\left(Z < \dfrac{24-\mu}{\sigma}\right) = 0.8$	
$\dfrac{24-\mu}{\sigma} = 0.8416212335$	(A1) for correct value
$\mu + 0.8416212335\sigma = 24 \ldots(2)$	A1
$(1) = (2)$	(M1) for setting equation
Solving, we have $\mu = 21.53555538$ and $\sigma = 2.928211076$.	
$\therefore \mu = 21.5$, $\sigma = 2.93$	A2 N4

[8]

4. $P(180 < X < \mu) = 0.1$

$P\left(Z < \dfrac{180-\mu}{\sigma}\right) = 0.4$	(M1) for standardization
$\dfrac{180-\mu}{\sigma} = -0.2533471011$	(A1) for correct value
$\mu - 0.2533471011\sigma = 180 \ldots(1)$	A1
$P(X < \mu + \sigma) - P(X < 192) = 0.09$	
$0.8413447404 - P(X < 192) = 0.09$	
$P(X < 192) = 0.7513447404$	
$P\left(Z < \dfrac{192-\mu}{\sigma}\right) = 0.7513447404$	
$\dfrac{192-\mu}{\sigma} = 0.6787275298$	(A1) for correct value
$\mu + 0.6787275298\sigma = 192 \ldots(2)$	A1
$(1) = (2)$	(M1) for setting equation
Solving, we have $\mu = 183.2617187$ and $\sigma = 12.87450554$.	
$\therefore \mu = 183$, $\sigma = 12.9$	A2 N4

[8]

Exercise 94

1.	$\mathrm{P}(X < Q_1) = 0.25$	(M1) for valid approach
	$Q_1 = 229.882656$	(A1) for correct value
	$\mathrm{P}(X < Q_3) = 0.75$	(M1) for valid approach
	$Q_3 = 250.117344$	(A1) for correct value
	The interquartile range of X	
	$= Q_3 - Q_1$	(A1) for correct formula
	$= 250.117344 - 229.882656$	
	$= 20.234688$	
	$= 20.2$	A1 N3
		[6]
2.	$\mathrm{P}(X < q_{30}) = 0.3$	(M1) for valid approach
	$q_{30} = 155.804797$	(A1) for correct value
	$\mathrm{P}(X < q_{70}) = 0.7$	(M1) for valid approach
	$q_{70} = 164.195204$	(A1) for correct value
	$q_{70} - q_{30}$	(A1) for correct formula
	$= 164.195204 - 155.804797$	
	$= 8.390407$	
	$= 8.39$	A1 N3
		[6]
3.	$\mathrm{P}(X < q_{90}) = 0.9$	(M1) for valid approach
	$q_{90} = 93.126207$	(A1) for correct value
	$\mathrm{P}(X < q_{10}) = 0.1$	(M1) for valid approach
	$q_{10} = 82.87379373$	(A1) for correct value
	$q_{90} - q_{10}$	(A1) for correct formula
	$= 93.126207 - 82.87379373$	
	$= 10.25241327$	
	$= 10.3$	A1 N3
		[6]
4.	$\mathrm{P}(X < s) = \frac{1}{3}$	(M1) for valid approach
	$s = 48.707818$	(A1) for correct value
	$\mathrm{P}(X < t) = \frac{2}{3}$	(M1) for valid approach
	$t = 51.292182$	(A1) for correct value
	$t - s$	
	$= 51.292182 - 48.707818$	(A1) for substitution
	$= 2.584364$	
	$= 2.58$	A1 N3
		[6]

Exercise 95

1. (a) $P(X > 102)$

$= 0.5 - P(\mu < X < 102)$ (M1) for valid approach

$= 0.5 - 0.45$

$= 0.05$ A1 N2

[2]

(b) $P(X < 102) = 0.95$

$P\left(Z < \frac{102-\mu}{2}\right) = 0.95$ (M1) for standardization

$\frac{102-\mu}{2} = 1.644853626$ A1

$\mu = 98.71029275$

$\mu = 98.7$ A1 N3

[3]

(c) $P((X < 102) \cap (Y < 102)) = 0.475$

$P(X < 102) \cdot P(Y < 102) = 0.475$ (M1) for independent events

$0.95 P(Y < 102) = 0.475$ (M1)(A1) for substitution

$P(Y < 102) = 0.5$ A1

$\therefore \lambda = 102$ A1 N3

[5]

(d) $P(Y < 100 \mid Y < 102)$

$= \frac{P(Y < 100 \cap Y < 102)}{P(Y < 102)}$ (M1) for valid approach

$= \frac{P(Y < 100)}{0.5}$ (A2) for correct values

$= \frac{0.252492467}{0.5}$ (A1) for correct value

$= 0.504984934$

$= 0.505$ A1 N3

[5]

2. (a) $P(X < 70)$

$= 1 - P(X > 70)$ (M1) for valid approach

$= 1 - 0.78$

$= 0.22$ A1 N2

[2]

(b) $P(X < 70) = 0.22$

$P\left(Z < \frac{70 - \mu}{4.5}\right) = 0.22$ (M1) for standardization

$\frac{70 - \mu}{4.5} = -0.7721932195$ A1

$\mu = 73.47486949$

$\mu = 73.5$ A1 N3

[3]

(c) $P((X < 70) \cap (Y < 70)) = 0.0484$

$P(X < 70) \cdot P(Y < 70) = 0.0484$ (M1) for independent events

$0.22\,P(Y < 70) = 0.0484$ (M1) for substitution

$P(Y < 70) = 0.22$ A1

$P\left(Z < \frac{70 - 80}{\sigma}\right) = 0.22$ (M1) for standardization

$\frac{-10}{\sigma} = -0.7721932195$

$\sigma = 12.95012666$

$\sigma = 13.0$ A1 N3

[5]

(d) $P(Y > 67 \mid Y < 70)$

$= \frac{P(Y > 67 \cap Y < 70)}{P(Y < 70)}$ (M1) for valid approach

$= \frac{P(67 < Y < 70)}{0.22}$ (A2) for correct values

$= \frac{0.0622747469}{0.22}$ (A1) for correct value

$= 0.2830670314$

$= 0.283$ A1 N3

[5]

3. (a) $P(X < 10)$
$= 0.5 - P(10 < X < 15)$
$= 0.5 - P(15 < X < 20)$ (M1) for valid approach
$= 0.5 - 0.35$
$= 0.15$ A1 N2

[2]

(b) $P(X < 10) = 0.15$

$P\left(Z < \frac{10-15}{\sigma}\right) = 0.15$ (M1) for standardization

$\frac{-5}{\sigma} = -1.03643338$ A1

$\sigma = 4.82423675$
$\sigma = 4.82$ A1 N3

[3]

(c) $P((X < 10) \cap (Y > 10)) = 0.075$
$P(X < 10) \cdot P(Y > 10) = 0.075$ (M1) for independent events
$0.15 P(Y > 10) = 0.075$ (M1)(A1) for substitution
$P(Y > 10) = 0.5$ A1
$\therefore \lambda = 10$ A1 N3

[5]

(d) $P(11 < Y < 12 | Y > 10)$

$= \frac{P(11 < Y < 12 \cap Y > 10)}{P(Y > 10)}$ (M1) for valid approach

$= \frac{P(11 < Y < 12)}{0.5}$ (A2) for correct values

$= \frac{0.1530794224}{0.5}$ (A1) for correct value

$= 0.3061588448$
$= 0.306$ A1 N3

[5]

4. (a) $\mathrm{P}(X < 240) = 0.5 + 0.4$

$\mathrm{P}(X < 240) = 0.9$

$\mathrm{P}\left(Z < \dfrac{240-200}{\sigma}\right) = 0.9$ (M1) for standardization

$\dfrac{40}{\sigma} = 1.281551567$ A1

$\sigma = 31.21216582$

$\sigma = 31.2$ A1 N3

[3]

(b) $\mathrm{P}((X > 240) \cap (Y > 240)) = 0.01$

$\mathrm{P}(X > 240) \cdot \mathrm{P}(Y > 240) = 0.01$ (M1) for independent events

$0.1\mathrm{P}(Y > 240) = 0.01$ (A1) for substitution

$\mathrm{P}(Y > 240) = 0.1$

$\mathrm{P}\left(Z < \dfrac{240-\lambda}{s}\right) = 0.9$ (M1) for standardization

$\dfrac{240-\lambda}{s} = 1.281551567$

$\lambda + 1.281551567s = 240 \ldots(1)$ A1

$\mathrm{P}((200 < X < 240) \cap (200 < Y < 240)) = 0.2$

$\mathrm{P}(200 < X < 240) \cdot \mathrm{P}(200 < Y < 240) = 0.2$

$0.4\mathrm{P}(200 < Y < 240) = 0.2$

$\mathrm{P}(200 < Y < 240) = 0.5$

$\mathrm{P}(Y < 200) = 1 - (0.5 + 0.1)$

$\mathrm{P}(Y < 200) = 0.4$

$\mathrm{P}\left(Z < \dfrac{200-\lambda}{s}\right) = 0.4$ (M1) for standardization

$\dfrac{220-\lambda}{s} = -0.2533471011$

$\lambda - 0.2533471011s = 220 \ldots(2)$ A1

Solving, we have $\lambda = 206.6023124$,

$s = 26.06035343$.

$\therefore \lambda = 207$, $s = 26.1$ A2 N5

[8]

(c) $\mathrm{P}(Y < 220 \mid 200 < Y < 240)$

$= \dfrac{\mathrm{P}(Y < 220 \cap 200 < Y < 240)}{\mathrm{P}(200 < Y < 240)}$ (M1) for valid approach

$= \dfrac{\mathrm{P}(200 < Y < 220)}{0.5}$ (A2) for correct values

$= \dfrac{0.2964096868}{0.5}$ (A1) for correct value

$= 0.5928193736$

$= 0.593$ A1 N3

[5]

Exercise 96

1. (a) (i) Let W be the weight of a randomly selected fish.

The required probability

$=\text{P}(W>850)$ — (M1) for valid approach

$=0.0083943057$

$=0.00839$ — A1 N2

(ii) $\text{P}(W>900\,|\,W>850)$ — (R1) for correct probability

$=\dfrac{\text{P}(W>850\cap W>900)}{\text{P}(W>850)}$ — (A1) for correct formula

$=\dfrac{\text{P}(W>900)}{\text{P}(W>850)}$

$=\dfrac{0.000252385136}{0.0083943057}$ — (A1) for correct values

$=0.0300662312$

$=0.0301$ — A1 N3

[6]

(b) The required probability

$=\text{P}(W>850)\times\text{P}(W>850)$ — (M1) for valid approach

$=0.0083943057\times0.0083943057$

$=0.00007046436819$

$=0.0000705$ — A1 N2

[2]

(c) (i) The required expected number

$=0.0083943057\times100$ — (A1) for correct formula

$=0.83943057$

$=0.839$ — A1 N2

(ii) Let X : Number of big fish in the selected sample

$X\sim B(100,\,0.0083943057)$ — (R1) for binomial distribution

The required probability

$=\text{P}(X>2)$ — (M1) for valid approach

$=1-\text{P}(X\le 2)$ — (A1) for correct value

$=0.0524471548$

$=0.0524$ — A1 N2

[6]

2. (a) (i) Let X be the volume of a randomly selected milk soda.

The required probability

$= \text{P}(X < 335)$ (M1) for valid approach

$= 0.0062096799$

$= 0.00621$ A1 N2

(ii) $\text{P}(X > 330 \mid X < 335)$ (R1) for correct probability

$= \dfrac{\text{P}(X > 330 \cap X < 335)}{\text{P}(X < 335)}$ (A1) for correct formula

$= \dfrac{\text{P}(330 < X < 335)}{\text{P}(X < 335)}$

$= \dfrac{0.00578061}{0.00620967}$ (A1) for correct values

$= 0.9309045408$

$= 0.931$ A1 N3

[6]

(b) The required probability

$= 2 \times \text{P}(X < 335) \times (1 - \text{P}(X < 335))$ (M1) for valid approach

$= 2 \times 0.00620967 \times (1 - 0.00620967)$ (A1) for substitution

$= 0.01234222$

$= 0.0123$ A1 N2

[3]

(c) (i) The required expected number

$= 0.00620967 \times 60$ (A1) for correct formula

$= 0.3725802$

$= 0.373$ A1 N2

(ii) Let X : Number of required milk soda

$X \sim B(60, 0.00620967)$ (R1) for binomial distribution

The required probability

$= \text{P}(X < 3)$ (M1) for valid approach

$= \text{P}(X \le 2)$ (A1) for correct value

$= 0.9937046328$

$= 0.994$ A1 N2

[6]

3. (a) (i) $P(L < t) = 0.15$ (M1) for valid approach

$t = 66.89069986$

$t = 66.9$ A1 N2

(ii) $P(L < 65 \mid L < t)$ (R1) for correct probability

$= \dfrac{P(L < 65 \cap L < t)}{P(L < t)}$ (A1) for correct formula

$= \dfrac{P(L < 65)}{P(L < t)}$

$= \dfrac{0.0477903304}{0.15}$ (A1) for correct values

$= 0.3186022024$

$= 0.319$ A1 N3

[6]

(b) The required probability

$= 2 \times P(L < t) \times (1 - P(L < t))$ (M1) for valid approach

$= 2 \times 0.15 \times (1 - 0.15)$ (A1) for substitution

$= 0.255$ A1 N2

[3]

(c) (i) The variance

$= 25 \times 0.15 \times (1 - 0.15)$ (A1) for correct formula

$= 3.1875$ A1 N2

(ii) $X \sim B(25, 0.15)$ (R1) for binomial distribution

The required probability

$= P(X \geq 4)$ (M1) for valid approach

$= 1 - P(X \leq 3)$ (A1) for correct value

$= 0.5288787147$

$= 0.529$ A1 N2

[6]

4. (a) (i) Let W be the weight of watermelons

$\text{P}(W > t) = 0.1$ — (M1) for valid approach

$t = 9.512620627$

$t = 9.51$ — A1 N2

(ii) $\text{P}(W < 9.8 | W > t)$ — (R1) for correct probability

$= \dfrac{\text{P}(W < 9.8 \cap W > t)}{\text{P}(W > t)}$ — (A1) for correct formula

$= \dfrac{\text{P}(t < W < 9.8)}{\text{P}(W > t)}$

$= \dfrac{0.0772500031}{0.1}$ — (A1) for correct values

$= 0.772500031$

$= 0.773$ — A1 N3

[6]

(b) The required probability

$= \text{P}(W > t) \times \text{P}(W > t) \times \text{P}(W > t)$ — (M1) for valid approach

$= 0.001$ — A1 N2

[2]

(c) (i) The variance

$= 52 \times 0.1 \times (1 - 0.1)$ — (A1) for correct formula

$= 4.68$ — A1 N2

(ii) $X \sim B(52, 0.1)$ — (R1) for binomial distribution

The required probability

$= \text{P}(13 \le X \le 26)$ — (M1) for valid approach

$= \text{P}(X \le 26) - \text{P}(X \le 12)$ — (A1) for correct value

$= 0.0014868739$

$= 0.00149$ — A1 N2

[6]

Chapter 22 Solution

Exercise 97

1.	(a)	(i)	$a = 0.2$	A1	N1
			$b = 52.4$	A1	N1
		(ii)	The estimated final exam score		
			$= 0.2(85) + 52.4$	(A1) for substitution	
			$= 69.4$	A1	N2
					[4]
	(b)	(i)	$r = 0.1832541665$		
			$r = 0.183$	A1	N1
		(ii)	Weak, Positive	A2	N2
					[3]
2.	(a)	(i)	$a = -2.085714286$		
			$a = -2.09$	A1	N1
			$b = 96.0952381$		
			$b = 96.1$	A1	N1
		(ii)	The estimated temperature		
			$= -2.085714286(9) + 96.0952381$	(A1) for substitution	
			$= 77.32380953$		
			$= 77.3^{\circ}\text{C}$	A1	N2
					[4]
	(b)	(i)	$r = -0.6074200776$		
			$r = -0.607$	A1	N1
		(ii)	Moderate, Negative	A2	N2
					[3]

3. (a) (i) $a = 0.7121409922$
$a = 0.712$ A1 N1
$b = 7.222584856$
$b = 7.22$ A1 N1

(ii) The estimated Physics test score
$= 0.7121409922(25) + 7.222584856$ (A1) for substitution
$= 25.02610966$
$= 25.0$ A1 N2

[4]

(b) -0.989 A1 N1

[1]

(c) The estimated difference
$= 0.18(3)$ (M1) for valid approach
$= 0.54$ A1 N2

[2]

4. (a) (i) $r = -0.9565269783$
$r = -0.957$ A1 N1

(ii) $a = -0.7459677419$
$a = -0.746$ A1 N1
$b = 6.748252688$
$b = 6.75$ A1 N1

[3]

(b) 0.178 A1 N1

[1]

(c) The estimated average number of hours
$= 0.53(2.7)$ (A1) for substitution
$= 1.431$ hours A1 N2

[2]

Exercise 98

1.	(a)	(i)	$a = 0.1566210046$		
			$a = 0.157$	A1	N1
			$b = -5.752968037$		
			$b = -5.75$	A1	N1
		(ii)	a represents the average increase of university entrance mark when the public exam score is increased by 1.	A1	N1
					[3]
	(b)		The estimated university entrance mark		
			$= 0.1566210046(180) - 5.752968037$	(A1) for substitution	
			$= 22.43881279$		
			$= 22.4$	A1	N2
					[2]
2.	(a)	(i)	$a = 3.422857143$		
			$a = 3.42$	A1	N1
			$b = 1.553333333$		
			$b = 1.55$	A1	N1
		(ii)	b represents the expected sales in 2011.	A1	N1
					[3]
	(b)		The estimated sales		
			$= 3.422857143(2.5) + 1.553333333$	(A1) for substitution	
			$= 10.11047619$		
			$= 10.1$ million dollars	A1	N2
					[2]
3.	(a)	(i)	$a = 5.978021978$		
			$a = 5.98$	A1	N1
			$b = 21.58241758$		
			$b = 21.6$	A1	N1
		(ii)	a represents the average increase of number of visitors when the maximum temperature is increased by 1 degree Celsius.	A1	N1
			b represents the expected number of visitors when the maximum temperature is zero degree Celsius.	A1	N1
					[4]
	(b)		The estimated number of visitors		
			$= 5.978021978(4) + 21.58241758$	(A1) for substitution	
			$= 45.49450549$		
			$= 45.5$	A1	N2
					[2]

4.	(a)	(i)	$a = 6.845588235$			
			$a = 6.85$	A1	N1	
			$b = 24.29338235$			
			$b = 24.3$	A1	N1	
		(ii)	a represents the average increase of the hardness of a metal ingot when its breaking strength is increased by 1 tonne per cm.	A1	N1	
			b represents the hardness of a metal ingot when its breaking strength is zero tonne per cm.	A1	N1	
						[4]
	(b)	The estimated hardness				
		$= 6.845588235(6) + 24.29338235$		(A1) for substitution		
		$= 65.36691176$				
		$= 65.4$		A1	N2	
						[2]

Exercise 99

1.	(a)	$a = -1.83140966$			
		$a = -1.83$	A1	N1	
		$b = 2164.965538$			
		$b = 2160$	A1	N1	
					[2]
	(b)	$e = 995$	M1A1	N2	
		$f = 638.8333333$			
		$f = 639$	A1	N1	
					[3]
2.	(a)	$a = 1.231628454$			
		$a = 1.23$	A1	N1	
		$b = -2.366255144$			
		$b = -2.37$	A1	N1	
					[2]
	(b)	$m = 6.6$	M1A1	N2	
		$n = 7.28$	A1	N1	
					[3]
3.	(a)	$a = -0.0003105590062$			
		$a = -0.000311$	A1	N1	
		$b = 7.541614907$			
		$b = 7.54$	A1	N1	
					[2]
	(b)	1	A1	N1	
					[1]
	(c)	$d = 69.8203125$	M1		
		$d = 69.8$	A1	N2	
					[2]
4.	(a)	$a = 0.2785493827$			
		$a = 0.279$	A1	N1	
		$b = 66.17052469$			
		$b = 66.2$	A1	N1	
					[2]
	(b)	1	A1	N1	
					[1]
	(c)	$x = 67$	A2	N2	
					[2]

Exercise 100

1.	(a)	(i)	$r = 0.8597409868$	(M1) for valid approach
			$r = 0.860$	A1 N2
		(ii)	$a = 0.0036032243$	
			$a = 0.00360$	A1 N1
			$b = -0.6258602021$	
			$b = -0.626$	A1 N1
				[4]
	(b)		The estimated monthly honey production	
			$= 0.0036032243(700) - 0.6258602021$	(A1) for substitution
			$= 1.896396808$	(A1) for correct value
			$= 1.9$ kg	A1 N3
				[3]
	(c)		The monthly honey production	
			$= 1.896396808 \times (1 + 2\%)^{12}$	(M1)(A1) for substitution
			$= 1.896396808 \times 1.02^{12}$	(A1) for simplification
			$= 2.405089691$	
			$= 2.41$ kg	A1 N2
				[4]
	(d)		$1.896396808 \times (1 + 2\%)^t = 3$	(M1) for setting equation
			$1.896396808 \times 1.02^t - 3 = 0$	(A1) for simplification
			$t = 23.161402$	(A1) for correct value
			Thus, the year is 2019.	A1 N2
				[4]

2.	(a)	(i)	$r = 0.9822040739$	(M1) for valid approach
			$r = 0.982$	A1 N2
		(ii)	$a = 2.5625$	A1 N1
			$b = 6.375$	A1 N1
				[4]
	(b)	The estimated monthly honey production		
		$= 2.5625(24) + 6.375$		(A1) for substitution
		$= 67.875$		(A1) for correct value
		$= 68$ kg		A1 N3
				[3]
	(c)	The monthly consumption of chicken food		
		$= 67.875 \times (1 + 5\%)^6$		(M1)(A1) for substitution
		$= 67.875 \times 1.05^6$		(A1) for simplification
		$= 90.95899161$		
		$= 91.0$ kg		A1 N2
				[4]
	(d)	$67.875 \times (1 + 5\%)^t = 100$		(M1) for setting equation
		$67.875 \times 1.05^t - 100 = 0$		(A1) for simplification
		$t = 7.9422239$		(A1) for correct value
		Thus, the time is February 2019.		A1 N2
				[4]

3.	(a)	(i)	$r = 0.9823629148$	(M1) for valid approach
			$r = 0.982$	A1 N2
		(ii)	$a = 14.06320542$	
			$a = 14.1$	A1 N1
			$b = 188.3205418$	
			$b = 188$	A1 N1
				[4]
	(b)	The estimated number of wolves		
		$= 14.06320542(11) + 188.3205418$		(A1) for substitution
		$= 343.0158014$		(A1) for correct value
		$= 343$		A1 N3
				[3]
	(c)	$f(10) = 930$		(M1) for setting equation
		$930 = 50(e^{0.01k(10)} + 2)$		(A1) for substitution
		$18.6 = e^{0.1k} + 2$		
		$e^{0.1k} = 16.6$		
		$0.1k = \ln 16.6$		
		$k = 28.09402695$		
		$k = 28.1$		A1 N2
				[3]
	(d)	$14.06320542t + 188.3205418 = 50(e^{0.01(28.09402695)t} + 2)$		(M1) for setting equation
		$14.06320542t + 188.3205418 = 50e^{0.2809402695t} + 100$		(A1) for correct working
		$50e^{0.2809402695t} - 14.06320542t - 88.3205418 = 0$		
		$t = 3.6593917$		(A1) for correct value
		Thus, the year is 1984.		A1 N2
				[4]

4. (a) (i) $r = -0.925877311$ — (M1) for valid approach

$r = -0.926$ — A1 N2

(ii) $a = -1.172413793$

$a = -1.17$ — A1 N1

$b = 58.75862069$

$b = 58.8$ — A1 N1

[4]

(b) The estimated number of breaths per minute

$= -1.172413793(12) + 58.75862069$ — (A1) for substitution

$= 44.68965517$ — (A1) for correct value

$= 45$ — A1 N3

[3]

(c) $v(8) = 75$ — (M1) for setting equation

$75 = \frac{10}{e^{8k}} + 70$ — (A1) for substitution

$5 = \frac{10}{e^{8k}}$

$e^{8k} = 2$

$8k = \ln 2$

$k = 0.0866433976$

$k = 0.0866$ — A1 N2

[3]

(d) $\frac{10}{e^{0.0866433976t}} + 70$ — (M1) for setting equation

$= 1.5(-1.172413793t + 58.75862069)$

$\frac{10}{e^{0.0866433976t}} + 70 = -1.75862069t + 88.13793104$ — (A1) for correct working

$\frac{10}{e^{0.0866433976t}} + 1.75862069t - 18.13793104 = 0$

$t = 7.2902653$ — (A1) for correct value

Thus, the time is after 7.29 minutes. — A1 N2

[4]